1. Patients arrive at a hospital emergency room at a rate of two per hour. Assume that the
arrivals are modeled by a Poisson process. A doctor works a shift from 6 a.m. to 6 p.m.

   a) If the doctor has seen six patients by 8 a.m., what is the probability that the doctor will
      see a total of nine patients by 9 a.m.?
   b) What is the expected time between the arrival of successive patients? What is the
      probability that the time between successive arrivals will be more than one hour?
   c) What is the expected time after coming on duty that the doctor will see her first patient?
      What is the probability that she will see her first patient in fifteen minutes or less after
      coming on duty?
   d) What is the probability that the doctor will see her thirteenth patient before 1 p.m.?
   e) Of patients admitted to the emergency room, 14% are classified as “urgent.” What is
      the probability that the doctor will see more than six urgent patients during her shift?
   f) The hospital also has a walk-in clinic to handle minor problems. Patients arrive at this
      clinic at the rate of four per hour. What is the probability that the total number of
      patients arriving at both the emergency room and clinic from 6 a.m. until 12 noon will
      be greater than thirty?

2. When a power surge occurs on electric lines, it can damage a computer plugged into the line
   if the computer does not have a surge protector. There are various types of surges. “Tiny”
   surges occur at the rate of eight per hour, but cannot damage a computer. “Small” surges
   occur at the rate of one every 18 hours, and will damage an unprotected computer with
   probability .005. “Moderate” surges occur at the rate of one every forty-six hours, and will
   damage an unprotected computer with probability .08. Suppose that the arrivals of each type
   of surge can be modeled by independent Poisson processes.

   a) What is the expected number of surges of any type during eight hours of computer
      work?
   b) What is the expected number of computer-damaging surges during eight hours of computer
      work?
   c) What is the probability that there will be no computer-damaging surges during eight
      hours of computer work?

3. Automobiles pass a point on a highway at a rate of one per minute. Suppose that 5% of all
   automobiles are trucks and the arrival process is well approximated by a Poisson process.

   a) What is the probability that at least one truck passes by during an hour?
b) Given that ten trucks have passed by in an hour, what is the expected total number of automobiles that have passed by in that time?

c) If fifty automobiles have passed by in an hour, what is the probability that five of them were trucks?

4. Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes with rates $\lambda_1$ and $\lambda_2$ respectively, measuring the number of customers arriving in stores 1 and 2 respectively.

a) What is the probability that a total of four customers arrive at the two stores combined in a one hour period?

b) Given that exactly four customers have arrived at the two stores combined, what is the probability that all four went to store 1?

c) Let $T$ denote the time of the first arrival at store 2. Then $N_1(T)$ is the number of arrivals at store 1 before the first arrival at store 2. Find the probability distribution of $N_1(T)$; that is, find $P(N_1(T) = k)$ for each $k \in \{0, 1, 2, 3, \ldots\}$.

5. Two individuals, $A$ and $B$, both require kidney transplants. If $A$ does not receive a new kidney, then she will die after an exponentially distributed time with rate $\mu_A$. Likewise, $B$ will die after an exponentially distributed time with rate $\mu_B$. New kidneys arrive in accordance with a Poisson process having rate $\lambda$. The first kidney will go to $A$ (or to $B$ if $B$ is alive and $A$ is not at that time) and the next one to $B$ (if $B$ is still alive at that time).

a) What is the probability that $A$ receives a new kidney?

b) What is the probability that $B$ receives a new kidney?

6. Cars cross a certain point in the highway in accordance with a Poisson process with rate $\lambda = 3$ per minute. Hillary runs blindly across the highway; it takes her $s$ seconds to cross.

a) Assume that if Hillary is on the highway when a car passes by, then she will be injured. What is the probability that she will cross the road uninjured? Do this calculation for $s = 2, s = 5, s = 10,$ and $s = 20$.

b) Assume that if Hillary is agile enough to escape from a single car, but that if she encounters two or more cars while crossing the road she will be injured. What is the probability that she will cross the road uninjured? Do this calculation for $s = 2, s = 5, s = 10,$ and $s = 20$.

7. Let $N(t)$ be a Poisson process with rate $\lambda$. Let $T_n$ denote the time of the $n^{th}$ arrival. Find

a) $E[T_4]$

b) $E[T_4|N(1) = 2]$

c) $E[N(4) - N(2)|N(1) = 3]$

8. Men and women enter a supermarket according to independent Poisson processes having respective rates two and four per minute. Starting at an arbitrary time, compute the probability that at least two men arrive before three women arrive.