MATH 520 Axioms for Incidence Geometry

A geometry satisfying the following four axioms is an *Incidence Geometry*.

Incidence Axiom 1.	For each two distinct points there exists a unique line on both of them.
Incidence Axiom 2.	For every line there exist at least two distinct points on it.
Incidence Axiom 3.	There exist at least three distinct points.
Incidence Axiom 4.	Not all points lie on the same line.

This set of axioms does not explicitly state that the number of points or lines is finite and applies to both finite (Four-Point, Fano's and Young's geometries) and infinite geometries (Euclidean for example)

	Four-Point Geo	Four-Line Geo	Fano's Geo	Young's Geo
Incidence Axiom 1				
Incidence Axiom 2				
Incidence Axiom 3				
Incidence Axiom 4				

Incidence Theorem 1.   If two lines intersect, then the intersection is exactly one point. (Proof by Contradiction)

Incidence Theorem 2.   For each point there exists at least two lines containing it. (Direct Proof)

Incidence Theorem 3.   There exist three lines that do not share a common point. (Direct Proof)

Existence of Parallel Lines

The incidence axioms do not explicitly state that parallel lines exists. Fano's geometry is an incidence geometry that does not contain parallel lines thus parallel lines cannot be deduced from the incidence axioms. Therefore, in order to prove the existence of parallel lines, additional axioms are needed (see Young's geometry Axiom 5)

If we consider any line *l* and any point *P* not on *l* then three alternatives exist.

1. There exist no lines on *P* parallel to *l*, or
2. There exists exactly one line on *P* parallel to *l*, or
3. There exists more than one line on *P* parallel to *l*.

An incidence geometry that assumes alternative 2 is *Euclidean*. *Non-Euclidean* geometries assume alternative 1 or 3 or these alternatives are implied by the axioms for that geometry.