The SMSG Postulates

In an attempt to generate a list of axioms that are more accessible to younger students the School Mathematics Study Group (SMSG) developed a system of axioms that was specifically designed for the use in high school geometry courses. These axioms, which do give rise to all theorems in Euclidean geometry, are not minimal in nature and are meant to move the student almost immediately to more interesting and less intuitively obvious results. The principle is this: if fewer axioms are proposed, then the more elementary (and obvious) results are needed to "start up" the theory of Euclidean geometry. The idea is to get younger students involved in more interesting results in a timely manner. So even though some of the SMSG axioms are redundant, they do achieve the desired effect of almost immediately being able to state significant results. Below is a list of the SMSG axioms, from which all of Euclidean geometry can be proven.

Undefined Terms: Point, line, and plane.

Postulate 1. (Line Uniqueness) Given any two different points, there is exactly one line which contains both of them.

Postulate 2 (Distance Postulate) To every pair of different points there corresponds a unique positive number.

Postulate 3 (Ruler Postulate) The points of a line can be placed in correspondence with the real numbers in such a way that (i) to every point of the line there corresponds exactly one real number called the point's *coordinate*, (ii) to every real number there corresponds exactly one point of the line, and (iii) The distance between two points is the absolute value of the difference of the corresponding coordinates.

Postulate 4 (Ruler Placement Postulate) Given two points P and Q of a line, the coordinate system can be chosen in such a way that the coordinate of P is zero and the corrdinate of Q is positive.

Postulate 5 (Points Exist) (a) Every plane contains at least three non-collinear points. (b) Space contains at least for non-coplanar points.

Postulate 6 (Points) If two points lie in a plane, then the line containing these points lies in the same plane.

Postulate 7 (Plane Uniqueness) Any three points lie in at least one plane, and any three non-collinear points lie in exactly one plane.

Postulate 8 (Plane Inersection) If two different planes intersect, then their intersection is a line.

Postulate 9 (Plane Separation Postulate) Given a line and a plane containing it, the points of the plane that do not lie on the line form two sets such that (i) each of the sets is convex, and (ii) if P is in one set and Q is in the other, then the segment $\overline{PQ}$ intersects the line.

Postulate 10 (Space Seperation Postulate) The points of space that do not lie in a given plane form two sets such that (i) each of the sets is convex, and (ii) if P is in one set and Q is in the other, then the segment $\overline{PQ}$ intersects the plane.

Postulate 11 (Angle Measurement Postulate) To every angle there corresponds a real number between 0° and 180°.

Postulate 12 (Angle Construction Postulate) Let $\overrightarrow{AB}$ be a ray on the edge of the half-plane H. For every r between 0 and 180 there is exactly one ray $\overrightarrow{AP}$ with P in H such that $m\angle PAB = r$.

Postulate 13 (Angle Addition Postulate) If D is a point in the interior of $m\angle BAC$, then $m\angle BAC = m\angle BAD + m\angle DAC$.

Postulate 14 (Supplementary Postulate) If two angles form a linear pair, then they are supplementary.

Postulate 15 (Side Angle Side Postulate) Given a one-to-one correspondence between two triangles (or between a triangle and itself), if two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, the the correspondence is a congruence.

Postulate16 (Parallel Postulate) Through a given external point there is at most one line parallel to a given line.

Postulate 17 To every polygonal region there corresponds a unique positive real number called its area.

Postulate 18 If two triangles are congruent, then the triangular regions have the same area.

Postulate 19 Suppose that the region R is the union of two regions R_1 and R_2. If R_1 and R_2 intersect at most in a finite number of segments and points, then the area of R is the sum of the areas of R_1 and R_2.

Postulate 20 The area of a rectangle is the product of the length of its base and the length of its altitude.

Postulate 21 The volume of a rectangular parallelpiped is equal to the product of the length of its altitude and the area of the base.

Postulate 22 (Cavalieri's Principle) Given two solids and a plane, if for every plane that intersects the solids and is parallel to the given plane the two intersections determine regions that have the same area, then the two solids have the same volume.