MATH 150 Logic Section 3.4: More on the Conditional

p: You run  q: You hide

Conditional p → q:

Converse q → p:

Inverse ~ p → ~ q:

Contrapositive ~ q → ~ p:

Examples
1) w: Tarzan lifts weights s: Tarzan sweats

Symbolically          In words
Conditional: ~ s → w  If Tarzan does not sweat, then Tarzan lifts weights.

Converse:

Inverse:

Contrapositive:

2) Conditional: ~ p → q
   Converse: ~ q → p
   Inverse: p → ~ q
   Contrapositive: q → ~ p

3) Conditional: ~ s → r
   Converse: ~ r → s
   Inverse: s → ~ r
   Contrapositive: r → ~ s

4) Statement: k → l
   Converse: l → k
   Inverse: ~ k → ~ l
   Contrapositive: ~ l → ~ k

Truth Table for conditional, converse, inverse, and contrapositive.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>~ p</th>
<th>~ q</th>
<th>p → q</th>
<th>q → p</th>
<th>~ p → ~ q</th>
<th>~ q → ~ p</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Therefore, p → q ≡ and q → p ≡

Other translations of p → q:

If p, then q. p implies q. p is sufficient for q.
If p, q. p only if q. q is necessary for p.
All p’s are q’s. q if p.
Write as “If... then...” statements.
1) You will be happy if you get an A.  
2) My toe hurts only if it is broken.
3) All snoops are snorks.  
4) Pigs flying is necessary for dogs to purr.

Write as a symbolic conditional. 

r: The sun shines  
s: I drink lemonade

1) The sun shines if I drink lemonade.  
2) The sun shines only if I drink lemonade.

The Biconditional:  

p if and only if q (p iff q). Symbolically this is written \( p \iff q \)

This means: If p, then q AND if q, then p. Symbolically:  

\[ p \iff q \equiv (p \rightarrow q) \land (q \rightarrow p) \]

Therefore, for a biconditional \( p \iff q \) to be true, it must be true in both directions; left to right and right to left.

Truth Table for the Biconditional

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>( p \rightarrow q )</th>
<th>( q \rightarrow p )</th>
<th>( (p \rightarrow q) \land (q \rightarrow p) )</th>
<th>( p \iff q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

Determine the truth value for each of the compound statements below:

1. Asia is a country \( \iff 6 + 4 = 10 \)
2. \(-3 < 0 \iff (p \lor \sim p)\)
3. \(1/3 \) is an integer \( \iff \pi \) is rational

Construct truth tables for:

4. \((p \land \sim q) \iff (q \rightarrow p)\)
5. \((\sim r \iff t) \lor (s \rightarrow r)\)