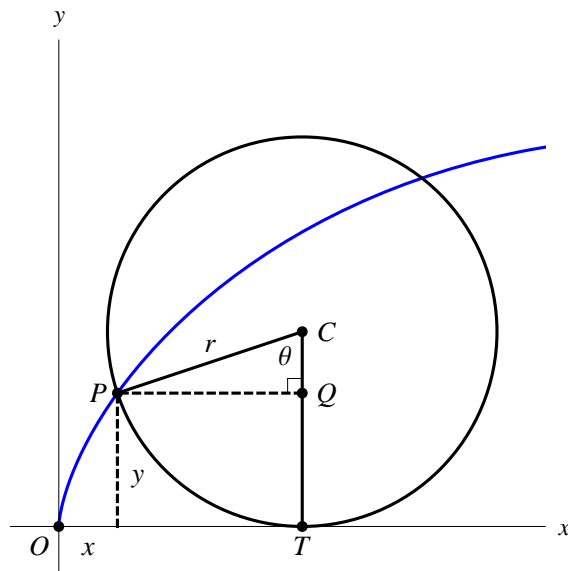


Laboratory 1
MATH 301 Honors

Rules: Answer the questions below. Please use a separate side of a piece of paper for each question. You may consult your text and use *Mathematica* (**but no other sources animate or inanimate**) for this test. You must show your work. If you use *Mathematica* to make a calculation, you must show the set up that leads to the calculation. Please direct any questions you may have to your instructor.

1. **The Cycloid** The cycloid is the curve generated by a point on a circle that rolls without slipping along a line. The Mathematica notebook accompanying this lab has an animation of the cycloid. Parametric equations can be found for the cycloid by referring to the following figure, which is also available in the Mathematica notebook. We must find the coordinates of P as functions of the parameter θ and the constant r , which is the radius of the rolling circle.



- a) Use the fact that the circle has rolled without slipping along the line to conclude that

$$|OT| = r\theta$$

and thus that the coordinates of C are $(r\theta, r)$.

- b) Let the coordinates of P be (x, y) . Show that $|PQ| = r \sin \theta$, and explain how this implies that

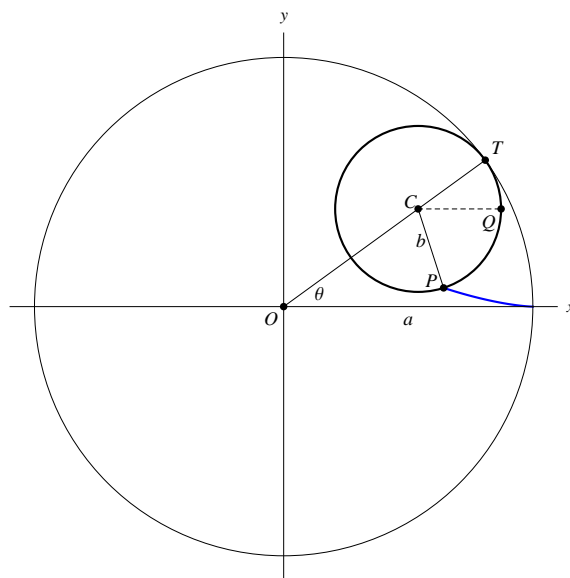
$$x = r\theta - r \sin \theta = r(\theta - \sin \theta)$$

- c) Show that $|QC| = r \cos \theta$, and explain how this implies that

$$y = r - r \cos \theta = r(1 - \cos \theta)$$

d) Show that the length of one arc of the cycloid is $8r$.

2. **The Hypocycloid** The hypocycloid is the curve generated by a point on a circle that rolls without slipping around the inside of a larger circle. The Mathematica notebook accompanying this lab has an animation of the hypocycloid. Parametric equations can be found for the hypocycloid by referring to the following figure, which is also available in the Mathematica notebook. We must find the coordinates of P as functions of the parameter θ , the radius a of the large circle, and the radius b of the smaller, rolling circle.



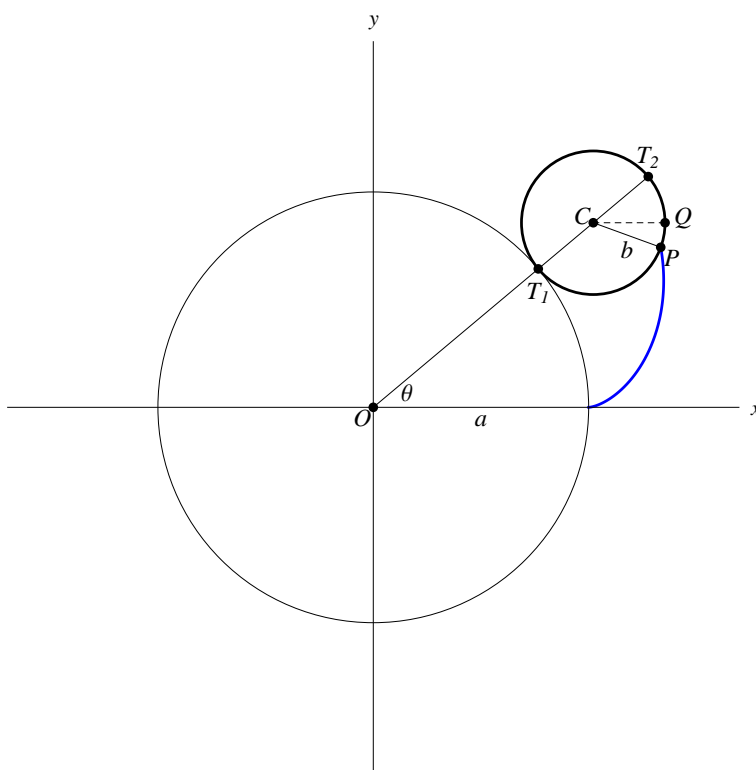
- Use the fact that the circle has rolled without slipping on the larger circle to conclude that the length of the circular arc TP is $a\theta$.
- Explain why the length of the circular arc TQ is $b\theta$, thus the length of the circular arc QP is $(a - b)\theta$.
- Show that the coordinates of C are $((a - b) \cos \theta, (a - b) \sin \theta)$.
- Use part b) to explain why the measure of the angle $\angle QCP$ is $\frac{a - b}{b}\theta$.
- Let the coordinates of P be (x, y) . Explain how parts b), c), and d) together imply that the coordinates of P are

$$x = (a - b) \cos \theta + b \cos \left(\frac{a - b}{b} \theta \right)$$

$$y = (a - b) \sin \theta - b \sin \left(\frac{a - b}{b} \theta \right)$$

- f) Use the accompanying Mathematica notebook to draw the graphs of hypocycloids with a a positive integer and $b = 1$. How does the value of a affect the graph?
- g) Now try $b = 1$ and $a = n/d$, a fraction where n and d have no common factor. First let $n = 1$ and try to determine the effect of the denominator d on the shape of the graph. Then let n vary while keeping d constant. What happens when $n = d + 1$?

3. **The Epicycloid** The epicycloid is the curve generated by a point on a circle that rolls without slipping around the outside of a larger circle. The Mathematica notebook accompanying this lab has an animation of the epicycloid. Parametric equations can be found for the epicycloid by referring to the following figure, which is also available in the Mathematica notebook. We must find the coordinates of P as functions of the parameter θ , the radius a of the large circle, and the radius b of the rolling circle.



- a) Let the coordinates of P be (x, y) . Follow an argument similar to that in Problem 2 to determine equations for x and y .
- b) Investigate graphs of epicycloids in a manner similar to parts f) and g) of Problem 2.