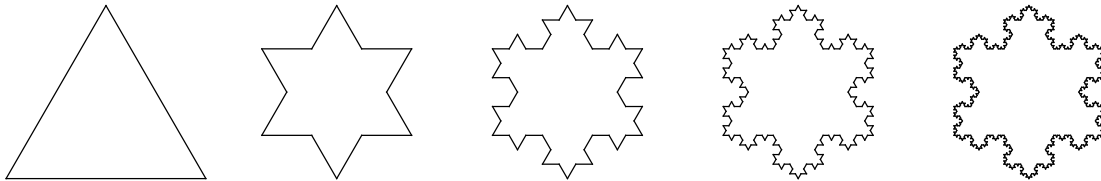


Laboratory 3
MATH 202 Honors

Rules: Answer the questions below. Please use a separate side of a piece of paper for each question. You may consult your text and use *Mathematica* (**but no other sources animate or inanimate**) for this test. You must show your work. If you use *Mathematica* to make a calculation, you must show the set up that leads to the calculation. Please direct any questions you may have to your instructor. The assignment is due at 3:00 p.m. on Wednesday, March 20.

1. To construct the **snowflake curve**, start with an equilateral triangle with sides of length 1. Step 1 in the construction is to divide each side into three equal parts, construct an equilateral triangle on the middle part, and then delete the middle part. Step 2 is to repeat step 1 for each side of the resulting polygon. This process is repeated at each succeeding step, and the snowflake curve is the curve that results from repeating this process indefinitely. The figure shows the original triangle and the first four steps in creating the snowflake curve



- Let s_k , l_k , and p_k be respectively the number of sides, the length of a side, and the total length (perimeter) of the k^{th} approximating curve; that is, the curve obtained after step k of the construction. Find formulas for s_k , l_k , and p_k .
- Show that $\lim_{k \rightarrow \infty} p_k = \infty$, so the snowflake curve has infinite length.
- The area A of an equilateral triangle with side length s is

$$A = \frac{s^2\sqrt{3}}{4}$$

Sum an infinite series to find the area enclosed by the snowflake curve. Thus the snowflake is infinitely long but encloses only a finite amount of area.

2. Find the sum of the series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{12} + \dots$$

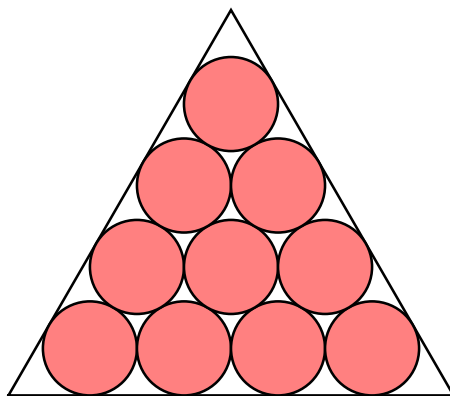
where the terms are the reciprocals of the positive integers whose only prime factors are 2's and 3's.

3. Suppose that circles of equal diameter are packed tightly in n rows inside an equilateral triangle. The figure below shows the case $n = 4$. If A is the area of the triangle and A_n is the total area occupied by the n rows of circles, find

$$\lim_{n \rightarrow \infty} \frac{A_n}{A}$$

Do the circles end up covering the entire triangle? If not, how much of the triangle will eventually be covered by the circles? **Hint:** It will be helpful to note that

$$\sum_{k=1}^n k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$



4. Here is an interesting problem that leads to an unproven conjecture known as the **hailstone conjecture**. Two different sequences are involved. First choose a positive integer N and call it a_0 . This value is called the *seed* of the *hailstone sequence* $\{a_n\}$. The rest of this sequence is generated as follows: For $n = 0, 1, 2, \dots$,

$$a_{n+1} = \begin{cases} a_n/2 & \text{if } a_n \text{ is even} \\ 3a_n + 1 & \text{if } a_n \text{ is odd} \end{cases}$$

However, if $a_n = 1$ for any n , then the sequence terminates.

- Compute the sequence that results from seeds $N = 2, 3, \dots, 10$. Verify that the sequence eventually terminates in all of these cases. The **hailstone conjecture** states that the sequence terminates after a finite number of terms for any seed N .
- Plot as many terms of the hailstone sequence as is feasible for as many seeds as you can. You might try developing *Mathematica* code to do this. How did the hailstone sequence get its name? Does the hailstone conjecture seem to be true?
- Define the sequence $\{b_k\}$ to be the number of terms needed for the sequence $\{a_n\}$ to terminate starting with a seed $N = k$. Verify that $b_2 = 1$, $b_3 = 7$, and $b_4 = 2$, and find b_k for all k less than or equal to 10.
- Plot as many terms in the sequence $\{b_n\}$ as you can, again employing *Mathematica* if possible.