Laboratory 2
MATH 202 Honors

Rules: Answer the questions below. Please use a separate side of a piece of paper for each question. You may consult your text and use *Mathematica* (**but no other sources animate or inanimate**) for this test. You must show your work. If you use *Mathematica* to make a calculation, you must show the set up that leads to the calculation. Please direct any questions you may have to your instructor. The assignment is due at 3:30 p.m. on Monday, March 4.

1. The Gamma function $\Gamma(x)$ is defined as

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$$

It can be shown that this improper integral converges for $x > 0$.

a) Find $\Gamma(1)$ and $\Gamma(2)$ by hand.

b) Prove that $\Gamma(x+1) = x\Gamma(x)$ for all $x > 0$, and use this identity to find $\Gamma(3)$ and $\Gamma(4)$.

c) Make a conjecture about $\Gamma(n)$ for n a positive integer.

d) It can be shown that

$$\int_0^\infty e^{-x^2}\, dx = \frac{1}{2}\sqrt{\pi}$$

Use this fact to find $\Gamma(\frac{1}{2})$, and use your value of $\Gamma(\frac{1}{2})$ to find $\Gamma(\frac{3}{2})$.

2. The speedometer in your car does not register the correct speed, but it does tell you how your speed is changing. You are travelling down the interstate and you need a way to estimate your speed. You decide to use your watch and the highway mile markers to time how long it takes you to travel a mile in seconds, using care to go at a constant speed for that interval.

a) Show that if x is number of seconds it takes to travel a mile, then your speed in miles per hour is

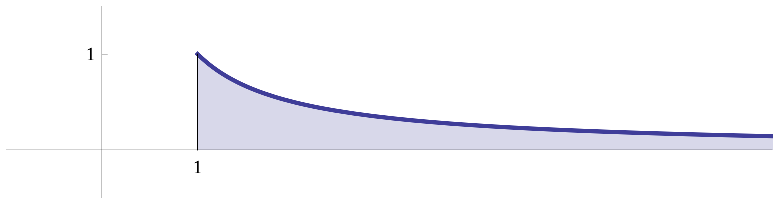
$$f(x) = \frac{3600}{x}$$

b) Note that $f(60) = 60$, so if it takes exactly sixty seconds to travel a mile then you are going 60 miles per hour. The formula for $f(x)$ is too difficult for you to calculate in your head, so you decide to approximate $f(x)$ using Taylor polynomials. Make a good choice for x_0 and find the first-degree Taylor polynomial $p_1(x)$ for $f(x)$ at $x = x_0$.

c) Use $p_1(x)$ to approximate your speed if it takes you 54 seconds to travel 1 mile. Compare this approximation with the actual value of $f(54)$.

d) Repeat parts b) and c) using $p_2(x)$.

3. Consider the region R bounded by the x-axis, the line $x = 1$ and the curve $y = \dfrac{1}{x}$, which is shown in the figure below.



a) Show that the region R has infinite area.

b) Revolve the region R about the x-axis to produce a solid. Show that the solid has finite volume.

c) Write down an improper integral than can be used to find the surface area of the solid in part b), then have Mathematica evaluate it. Does the solid have finite or infinite surface area?

d) Consider painting the region R, filling the solid with paint, and painting the exterior of the solid. Do the results from part a) through c) seem to be consistent?

4. Pharmacokinetics describes the process by which drugs are assimilated by the body. The elimination of most drugs from the body may be modeled by an exponential decay function $y(t) = y_0 e^{-kt}$ with a known half-life, where $y(t)$ is the amount of the drug in the blood. The simplest models assume that an entire drug dose is immediately absorbed into the blood.

a) Suppose that an initial dose of 100 mg of a drug is administered. The drug's half life is 5 hours. Find the exponential decay function that models the amount of drug in the blood t hours after it is administered.

b) Suppose that the maximum safe level of the drug in the blood is 130 mg. How long after the initial dose may another 100-mg dose be administered? How long after the initial dose may a 50-mg dose be administered?

c) If a second 100-mg dose is given 12 hours after the initial dose, how long after the second dose will the amount of drug in the blood reach 10 mg?

d) Devise a dosing regimen (how much in each dose and how long between doses) that would keep between 80 mg and 110 mg in the blood. Try to keep the regimen as simple as possible.