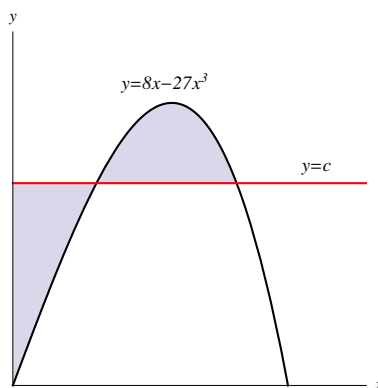


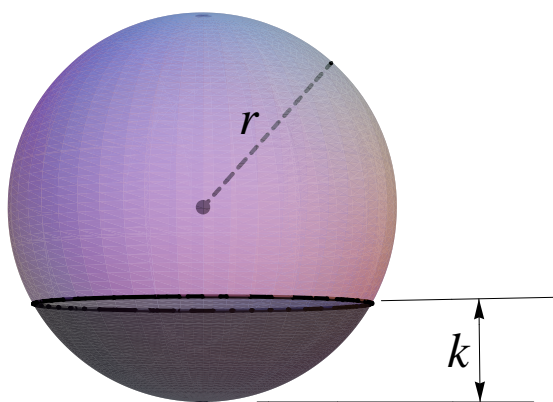
Laboratory 1
MATH 202 Honors

Rules: Answer the questions below. Please use a separate side of a piece of paper for each question. You may consult your text and use *Mathematica* (**but no other sources animate or inanimate**) for this test. You must show your work. If you use *Mathematica* to make a calculation, you must show the set up that leads to the calculation. Please direct any questions you may have to your instructor. The test is due at 5:00 p.m. on Monday, February 4.

1. The figure below shows the regions bounded by the line $y = c$ and the curve $y = 8x - 27x^3$. Find the value of c for which the areas of the shaded regions are equal. **Hint:** Consider drawing a line from the rightmost intersection point straight down to the x -axis and comparing areas.



2. A **spherical segment** is the region lying within a sphere and also lying on one side of a given plane that intersects the sphere (see the figure).



a) Show that the volume of a spherical segment of height k of a sphere of radius r is

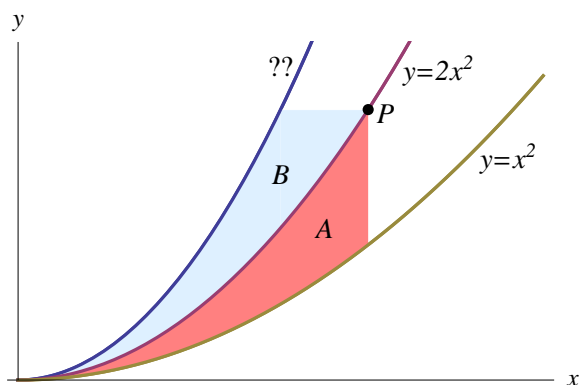
$$V = \frac{1}{3}\pi k^2(3r - k)$$

- b) Show that if a sphere of radius 1 is sliced into two pieces by a plane at a distance x from its bottom in such a way that the volume of the larger piece is three times the volume of the smaller, then x is a solution of the equation

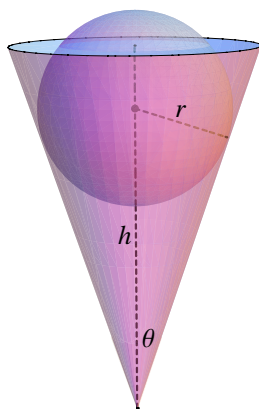
$$x^3 - 3x + 1 = 0$$

where $0 < x < 1$. Use Mathematica to approximate x .

3. The figure below shows three curves. The unknown curve has the property that, for every point P on the middle curve the areas A and B are equal. Find an equation for the unknown curve. **Hint:** It may help to first think of the unknown curve as being in the form $x = g(y)$.

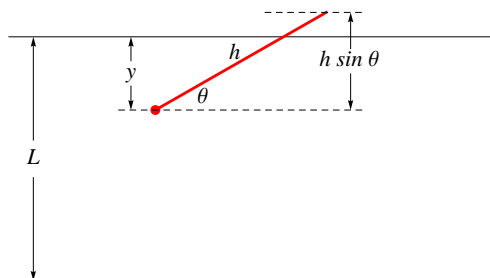


4. A conical cup filled with water has the shape of a cone with height h and semivertical angle θ (see the figure below). A ball of radius r is placed in the cup, and pushed down until it just touches the side of the cup, thus making some of the water from the cup overflow.

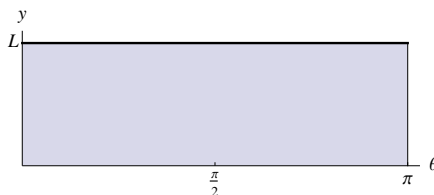


- a) Let $h = 1$ and $\theta = \pi/6$. Find the value of r that causes the greatest volume of water to overflow.
- b) Solve the same problem as in a) for arbitrary h and θ .

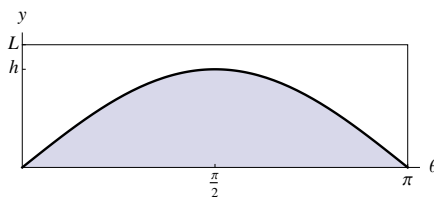
5. *Buffon's Needle Problem* A needle of length h is dropped onto a flat surface on which parallel lines have been drawn L units apart, where $L \geq h$. The problem is to determine the probability that the needle will come to rest intersecting one of the lines. Assume that the lines run parallel to the x -axis as in the figure. As shown in the figure, the needle will come to rest with its lower end a distance y from the line above it and at an angle θ with the horizontal. (If the needle lands parallel to the lines, let its left end be its lower end and let $\theta = 0$.)



Note that $0 \leq y \leq L$ and $0 \leq \theta \leq \pi$, and that all such values of y and θ are equally likely to happen if the needle is tossed “randomly.” The rectangle in the figure below is called the *sample space* for the experiment; each point in the sample space corresponds to a possible outcome of the experiment.



Note further from the first figure that a needle will intersect a line whenever $y < h \sin \theta$. Thus the set of outcomes for which the needle intersects a line is displayed in the following figure.



The proportion of times that a randomly tossed needle will intersect a line (that is, the probability that a needle intersects a line) is the ratio

$$\frac{\text{area under } y = h \sin \theta}{\text{area of rectangle}}$$

Find the probability that the needle will intersect a line in terms of h and L . What happens if $h = L$? What happens if $h = \frac{1}{2}L$?