

TAYLOR REMAINDER SUPPLEMENTARY PROBLEMS – KEY
MATH 202

1.
 - a. $p_2(x) = 1 + \frac{1}{2}(x - 1) - \frac{1}{8}(x - 1)^2$
 - b. $\sqrt{1.1} \approx 1 + \frac{1}{2}(.1) - \frac{1}{8}(.1)^2 = 1.04875$
 - c. $|f'''(x)| = |\frac{3}{8}x^{-5/2}| \leq \frac{3}{8}$ for $1 \leq x \leq 1.1$, so let $M = \frac{3}{8}$
Thus $|R_2(1.1)| \leq \frac{3/8}{3!}(.1)^3 = 0.0000625$
 - d. $1.0486875 \leq \sqrt{1.1} \leq 1.0488125$
 - e. According to Mathematica, $\sqrt{1.1} = 1.04880885$

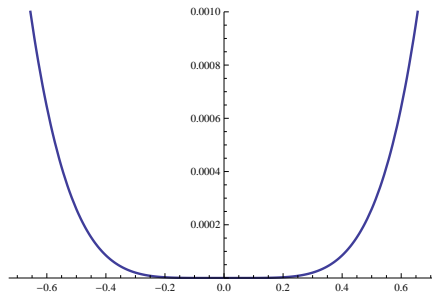
2.
 - a. $p_2(x) = \frac{1}{2} - \frac{1}{4}(x - 2) + \frac{1}{8}(x - 2)^2$
 - b. $\frac{1}{2.05} \approx \frac{1}{2} - \frac{1}{4}(.05) + \frac{1}{8}(.05)^2 = 0.4878125$
 - c. $|f'''(x)| = |6x^{-4}| \leq 6(\frac{1}{2})^4 = \frac{3}{8}$ for $2 \leq x \leq 2.05$, so let $M = \frac{3}{8}$
Thus $|R_2(2.05)| \leq \frac{3/8}{3!}(.05)^3 = 0.0000078125$
 - d. $0.4878046875 \leq \frac{1}{2.05} \leq 0.4878203125$
 - e. According to Mathematica, $\frac{1}{2.05} = 0.48780487804878053$

3.
 - a. $p_3(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$
 - b. $e^{-1/2} \approx 1 - (-\frac{1}{2}) + \frac{1}{2}(-\frac{1}{2})^2 - \frac{1}{6}(-\frac{1}{2})^3 = 0.604167$
 - c. $|f^{(4)}(x)| = e^{-x} \leq e^0 = 1$ for $0 \leq x \leq 1/2$, so let $M = 1$
Thus $|R_3(1/2)| \leq \frac{1}{4!}(\frac{1}{2})^4 = 0.002604167$
 - d. $0.6015625 \leq e^{-1/2} \leq 0.606770833$
 - e. According to Mathematica, $e^{-1/2} = 0.6065306597126334$

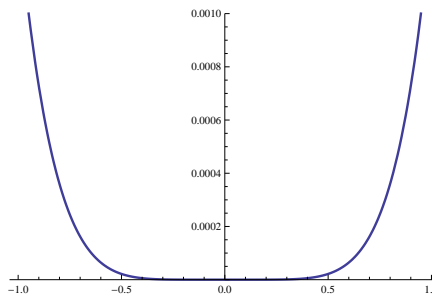
4.
 - a. $p_5(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$
 - b. $\cos(0.3) \approx 1 - \frac{1}{2}(.3)^2 + \frac{1}{24}(.3)^4 = 0.9553375$
 - c. $|f^{(6)}(x)| = |-\cos x| \leq 1$ for $0 \leq x \leq 0.3$, so let $M = 1$
Thus $|R_5(.3)| \leq \frac{1}{6!}(.3)^6 = 0.0000010125$
 - d. $0.9553364875 \leq \cos(0.3) \leq 0.9553385125$
 - e. According to Mathematica, $\cos(0.3) = 0.955336489125606$

5. Since $|f^{(n)}(.3)| \leq 1$, we let $M = 1$. Thus we set $|R_n(.3)| \leq \frac{1}{(n+1)!}(.3)^{n+1} \leq 0.001$ and solve for n to determine that $n = 3$.

6. Since $|f^{(n)}(.1)| \leq 1$, we let $M = 1$. Thus we set $|R_n(.1)| \leq \frac{1}{(n+1)!}(.1)^{n+1} \leq 0.001$ and solve for n to determine that $n = 2$.
7. Since $|f^{(n)}(.6)| = e^6 < e < 3$, we let $M = 3$. Thus we set $|R_n(.6)| \leq \frac{3}{(n+1)!}(.6)^{n+1} \leq 0.001$ and solve for n to determine that $n = 5$.
8. Since $|f^{(n)}(.3)| = e^3 < e < 3$, we let $M = 3$. Thus we set $|R_n(.3)| \leq \frac{3}{(n+1)!}(.3)^{n+1} \leq 0.001$ and solve for n to determine that $n = 4$.
9. Since $|f^{(5)}(x)| = |\cos x| \leq 1$, we let $M = 1$. Thus we set $|R_4(x)| \leq \frac{1}{5!}|x|^5 \leq 0.001$ and solve for x to determine that $|x| < 0.654389$, or $-0.654389 < x < 0.654389$.



10. Since $|f^{(6)}(x)| = |-\cos x| \leq 1$, we let $M = 1$. Thus we set $|R_5(x)| \leq \frac{1}{6!}|x|^6 \leq 0.001$ and solve for x to determine that $|x| < 0.946721$, or $-0.946721 < x < 0.946721$.



11. Since $|f^{(4)}(x)| = 16e^{-2x} \leq 16$ for $x > 0$, we let $M = 16$. Thus we set $|R_3(x)| \leq \frac{16}{4!}x^4 \leq 0.001$ and solve for x to determine that $x < 0.114471$, or $0 < x < 0.114471$.

