DERIVATIVES: RULES, FORMULAS, ETC.

Definition: The **derivative** of a function $f(x)$ at x is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

if this limit exists.

General Rules:

- $\dfrac{d}{dx}(a) = 0$, where a is a constant.
- $\dfrac{d}{dx}(ax) = a$, where a is a constant.
- $\dfrac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$
- $\dfrac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$
- $\dfrac{d}{dx}(x^n) = nx^{n-1}$, where $n \neq 0$. (Power Rule)
- $\dfrac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x)$ (Product Rule)
- $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$ (Quotient Rule)
- $\dfrac{d}{dx}(f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)
- $\dfrac{d}{dx}(f^{-1}(x)) = \dfrac{1}{f'(f^{-1}(x))}$ if f is invertible.

Formulas:

- $\dfrac{d}{dx}(\sin x) = \cos x$
- $\dfrac{d}{dx}(\cos x) = -\sin x$
- $\dfrac{d}{dx}(\tan x) = \sec^2 x$
- $\dfrac{d}{dx}(\cot x) = -\csc^2 x$

- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\csc x) = -\csc x \cot x$
- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
- $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ or $\frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$ or $-\frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(a^x) = a^x \ln a$, for $a > 0, a \neq 1$
- $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$, for $a > 0, a \neq 1$

Theorems:

Rolle's Theorem: If f is continuous on the interval $[a, b]$ and is differentiable on the interval (a, b) and $f(a) = f(b)$, then there exists a c in the interval (a, b) for which $f'(c) = 0$.

Mean Value Theorem: If f is continuous on the interval $[a, b]$ and is differentiable on the interval (a, b), then there exists a c in the interval (a, b) for which $f'(c) = \frac{f(b) - f(a)}{b - a}$.

L'Hôpital's Rule: Let $f(x)$ and $g(x)$ be functions differentiable on an open interval containing c (except possibly at c), with $g'(x) \neq 0$ on this open interval (except possibly at c). If $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$, then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

if this limit exists.