Chapter 2 Notes – math review

Relationships

Single functions Linear

Multivariable functions

Nonlinear

Within the functions; we are concerned with the relationship between the independent variable and the dependent variable.

If X (independent) changes, what happens to Y (dependent)?

From principles, we learned the concept of marginal analysis.

A one unit change in Q will cause Revenue to change by some amount… (this is known as marginal revenue.) Which variable is independent, which is dependent?

A one unit change in Labor will cause output (Q) to change by some amount… (this is known as the marginal product of labor).

Changes are elasticity’s.

To find these relationships in linear functions, we need to only look at the slopes.

Y=5+3X

Y=10+4I+3A+7X

On a typical demand curve, we can view the slope across ranges or at a specific point.

|  |  |  |
| --- | --- | --- |
| P | Q | TR |
| 2 | 20 | 40 |
| 3 | 18 | 56 |
| 4 | 16 | 64 |
| 5 | 14 | 74 |
| 6 | 12 | 72 |
| 7 | 10 | 70 |
| 8 | 8 | 64 |

74

TR

Q

P

MR

When the total revenue is at its highest point, the

Q

marginal revenue is zero. The marginal revenue

is the slope of the total revenue graph at each

quantity.

In non-linear function, the slope is different at each point.

In non-linear functions, we need to use the concept of a derivative.

A derivative is the slope of a function at a point. Or the rate of change in Y as X changes at some point along the function.

So if we want to find the marginal revenue, the change in TR when Q changes, we apply the derivate in a non-linear function.

1. TR = 50Q-1.5Q2
2. dTR/dQ = 50-3Q

The change in revenue based on Q, depends on what the value of Q is.



15

10

TR

1. When Q=10, the slope = 20; Q=15, slope=5
2. When the slope is zero, the total revenue function is at its peak.

We can find this point by setting the derivative of Total Revenue parabola equal to 0.

dTR/dQ = 0 =50-3Q Q=16.67 maximum revenue

Q= 16.67, slope=0, revenue is maximized

The rule for finding a derivative is Y=aX2  dY/dX=a\*n\*X(n-1)

So dTR/dQ=50Q(1-1)=50Q0=50

dTR/dQ= -1.5Q2= -3Q(2-1)= -3Q1= -3Q

other examples Y=10X-4X2 dY/dX=10-8X

Y= -10+3X2+6X3 dY/dX=6X+18X2

Graphically

Marginal revenue is the slope of the total revenue function.

Where MR=0, revenue is maximized.

MR

16.67

Q

16.67

10

Q

TR

TR

**The Profit Function**

 =TR-TC



Goal is to maximize profit 



Q

What is under our control? “Q”

Q\*

What is the slope of the  function when  is maximized? “0”

P=50-1.5Q

TR=50Q-1.5Q2

TC=5+6Q+0.5Q2

 =50Q-1.5Q2-5-6Q-0.5Q2

 =-5+44Q-2Q2

d /dQ=44-4Q=0

**Q=11**

TR=550-181.5=368.5

TC=5+66+60.5=131.5

 =237

237

Q

11

d /dQ=0



What if our function looked like?

qa

qb

Q



What point are we going to maximize qb

What is the slope of the  function at that point? 0

Where else is the slope of the function equal to 0?

At qa the quantity where  are minimized

When we solve for = 0, we will get two values of q.

We can’t assume that the higher value of q is optimal.

So we have use a 2nd derivative, which is a derivative of the first derivative of

the first derivative. To find a max and a min

A small increase in qa will increase 

A small increase in qb will decrease 

In our previous example

Q-2Q2

d/ d Q = -4 a negative is a maximum

if Q 

Q = 12 -5 + 44(12) -2 (144)

= 0.333 Q3 – 10 Q2 + 75Q +2

Q2 -20Q + 75 = 0

(Q-15)(Q-5) = 0

Q =15 Q = 5

d/ d Q = 2Q-20

If Q = 15, d/ d Q = 10 min

If Q = 5, d/ d Q = -10 max

max





 min





Multivariable Functions

Q= f (p, A)

What affect does price have on Q?

What affect does advertising have on Q?

= the change in output based on a change in price, holding a constant.

= ……….

Enrollment = f (A,T, I) the school can control A and T

Advertising, tuition, income I constant at 400

E’ = 591 + 400 – A2 + 20A – T2 + 6T

E = 991 – A2 + 20A – T2 + 6T + I

Find the optimal amount of advertising + tuition

A = $1000 of advertising

T = $100 for 3 hours of credit (per course)

A = 10 T = 3

Buy 10 units of advertising at $1,000 per unit

Charge 3 units per course at $100 per unit.

When doing a partial, treat other variables as if they were constants.

Y = 6A2B + 2 B + 5A

6A2 + 2

Summary of Chapter 2

The goal of the corporation is to maximize their profit.

We learned from econ 215 that profit is maximized when MR = MC

Let’s start with MR

TR = P\*Q

We are used to linear demand curves

P = a + bQ

a is the intercept b is the slope

if P = 24.00 Q = 0

if P = 22.50 Q = 1

if P = 21.00 Q = 2

as P falls Q rises – this is consistent with the law of demand

In this example the slope is -1.5

Plugging numbers into the P= a + bQ equation we can solve for the intercept

24 = a +b(0) therefore the intercept is 24

The linear demand curve is

P = 24 -1.5Q

TR = P\*Q

TR = 24Q – 1.5Q2

The marginal revenue is the derivative of the TR function

dTR/dQ – this represents the slope of the TR line

therefore

MR = 24 – 3Q

We learned from econ 215 that revenue is maximized when MR = 0 or elasticity = -1

If MR = 0

24-3Q = 0

Revenue is maximized when Q = 8

When Q = 8 TR = 96

TR = 24 (8) -1.5 (64)

TC = FC + VC

If FC = $8

and VC = $4Q+$.5Q2

then TC = 8 + 4Q+.5Q2

Cost are minimized where MC =AC

MC = dTC/dQ

AC = TC/Q

MC = 4 + Q

AC = 8/Q + 4 + .5Q

MC = AC

4 + Q = 8/Q + 4 + .5Q cancel out the 4s on both side

Q = 8/Q +.5Q subtract .5Q from both sides

.5Q = 8/Q cross multiply

.5Q2 = 8

Q2 = 16

Q = 4

Profit = TR – TC

Profit is maximized when MR = MC

This can be found by taking the derivative of the profit function and setting it equal to 0.

At that point marginal profit (the profit made by selling one more unit) is zero.

Profit = 24Q – 1.5Q2 - (8 + 4Q + .5Q2)

Or profit = -8+20Q-2Q2

Take the derivative and set it equal to 0

0 = 20 – 4Q

Q = 5

Plug 5 into the profit function for Q

Profit = 24Q – 1.5Q2 - (8 + 4Q + .5Q2)

Profit = $42