Interval Estimation of $\mu_1 - \mu_2$: $\sigma_1$ and $\sigma_2$ Known

- Example: Par, Inc.
  - Par, Inc. is a manufacturer of golf equipment and has developed a new golf ball that has been designed to provide “extra distance.”
  - In a test of driving distance using a mechanical driving device, a sample of Par golf balls was compared with a sample of golf balls made by Rap, Ltd., a competitor. The sample statistics appear on the next slide.
Interval Estimation of $\mu_1 - \mu_2$: $\sigma_1$ and $\sigma_2$ Known

Example: Par, Inc.

<table>
<thead>
<tr>
<th>Sample #1</th>
<th>Sample #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Par, Inc.</td>
<td>Rap, Ltd.</td>
</tr>
</tbody>
</table>

- Sample Size: 120 balls, 80 balls
- Sample Mean: 275 yards, 258 yards

Based on data from previous driving distance tests, the two population standard deviations are known with $\sigma_1 = 15$ yards and $\sigma_2 = 20$ yards.
Hypothesis Tests About $\mu_1 - \mu_2$: $\sigma_1$ and $\sigma_2$ Known

Example: Par, Inc.

Can we conclude, using $\alpha = .01$, that the mean driving distance of Par, Inc. golf balls is greater than the mean driving distance of Rap, Ltd. golf balls?
Interval Estimation of $\mu_1 - \mu_2$: $\sigma_1$ and $\sigma_2$ Known

- Example: Par, Inc.

Let us develop a 95% confidence interval estimate of the difference between the mean driving distances of the two brands of golf ball.
Difference Between Two Population Means: 
$\sigma_1$ and $\sigma_2$ Unknown

Example: Specific Motors

Specific Motors of Detroit has developed a new automobile known as the M car. 24 M cars and 28 J cars (from Japan) were road tested to compare miles-per-gallon (mpg) performance. The sample statistics are shown on the next slide.
Difference Between Two Population Means:
\( \sigma_1 \) and \( \sigma_2 \) Unknown

- Example: Specific Motors

<table>
<thead>
<tr>
<th>Sample #1</th>
<th>Sample #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>M Cars</td>
<td>J Cars</td>
</tr>
</tbody>
</table>
| 24 cars    | 28 cars    | Sample Size
| 29.8 mpg   | 27.3 mpg   | Sample Mean
| 2.56 mpg   | 1.81 mpg   | Sample Std. Dev.
Hypothesis Tests About $\mu_1 - \mu_2$: $\sigma_1$ and $\sigma_2$ Unknown

- Example: Specific Motors

Can we conclude, using a .05 level of significance, that the miles-per-gallon ($mpg$) performance of M cars is greater than the miles-per-gallon performance of J cars?
Inferences About the Difference Between Two Population Means: Matched Samples

- **Example: Express Deliveries**

  A Chicago-based firm has documents that must be quickly distributed to district offices throughout the U.S. The firm must decide between two delivery services, UPX (United Parcel Express) and INTEX (International Express), to transport its documents.
Inferences About the Difference Between Two Population Means: Matched Samples

Example: Express Deliveries

In testing the delivery times of the two services, the firm sent two reports to a random sample of its district offices with one report carried by UPX and the other report carried by INTEX. Do the data on the next slide indicate a difference in mean delivery times for the two services? Use a .05 level of significance.
### Inferences About the Difference Between Two Population Means: Matched Samples

<table>
<thead>
<tr>
<th>District Office</th>
<th>Delivery Time (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UPX</td>
</tr>
<tr>
<td>Seattle</td>
<td>32</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>30</td>
</tr>
<tr>
<td>Boston</td>
<td>19</td>
</tr>
<tr>
<td>Cleveland</td>
<td>16</td>
</tr>
<tr>
<td>New York</td>
<td>15</td>
</tr>
<tr>
<td>Houston</td>
<td>18</td>
</tr>
<tr>
<td>Atlanta</td>
<td>14</td>
</tr>
<tr>
<td>St. Louis</td>
<td>10</td>
</tr>
<tr>
<td>Milwaukee</td>
<td>7</td>
</tr>
<tr>
<td>Denver</td>
<td>16</td>
</tr>
</tbody>
</table>
Market Research Associates is conducting research to evaluate the effectiveness of a client’s new advertising campaign. Before the new campaign began, a telephone survey of 150 households in the test market area showed 60 households “aware” of the client’s product.

The new campaign has been initiated with TV and newspaper advertisements running for three weeks.
Hypothesis Tests about $p_1 - p_2$

- **Example: Market Research Associates**

  A survey conducted immediately after the new campaign showed 120 of 250 households “aware” of the client’s product.

  Can we conclude, using a .05 level of significance, that the proportion of households aware of the client’s product increased after the new advertising campaign?
Calculating the Probability of a Type II Error

- Example: Metro EMS (revisited)

  Recall that the response times for a random sample of 40 medical emergencies were tabulated. The sample mean is 13.25 minutes. The population standard deviation is believed to be 3.2 minutes.

  The EMS director wants to perform a hypothesis test, with a .05 level of significance, to determine whether or not the service goal of 12 minutes or less is being achieved.
Calculating the Probability of a Type II Error

1. Hypotheses are: \( H_0: \mu \leq 12 \) and \( H_a: \mu > 12 \)

2. Rejection rule is: Reject \( H_0 \) if \( z \geq 1.645 \)

3. Value of the sample mean that identifies the rejection region:

\[
z = \frac{\bar{x} - 12}{3.2 / \sqrt{40}} \geq 1.645
\]

\[
\bar{x} \geq 12 + 1.645 \left( \frac{3.2}{\sqrt{40}} \right) = 12.8323
\]

4. We will accept \( H_0 \) when \( \bar{x} < 12.8323 \)
Calculating the Probability of a Type II Error

5. Probabilities that the sample mean will be in the acceptance region:

<table>
<thead>
<tr>
<th>Values of $\mu$</th>
<th>$z = \frac{12.8323 - \mu}{3.2 / \sqrt{40}}$</th>
<th>$\beta$</th>
<th>$1 - \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.0</td>
<td>-2.31</td>
<td>.0104</td>
<td>.9896</td>
</tr>
<tr>
<td>13.6</td>
<td>-1.52</td>
<td>.0643</td>
<td>.9357</td>
</tr>
<tr>
<td>13.2</td>
<td>-0.73</td>
<td>.2327</td>
<td>.7673</td>
</tr>
<tr>
<td>12.8323</td>
<td>0.00</td>
<td>.5000</td>
<td>.5000</td>
</tr>
<tr>
<td>12.8</td>
<td>0.06</td>
<td>.5239</td>
<td>.4761</td>
</tr>
<tr>
<td>12.4</td>
<td>0.85</td>
<td>.8023</td>
<td>.1977</td>
</tr>
<tr>
<td>12.0001</td>
<td>1.645</td>
<td>.9500</td>
<td>.0500</td>
</tr>
</tbody>
</table>
Calculating the Probability of a Type II Error

- Calculating the Probability of a Type II Error

Observations about the preceding table:

- When the true population mean $\mu$ is close to the null hypothesis value of 12, there is a high probability that we will make a Type II error.

  Example: $\mu = 12.0001$, $\beta = .9500$

- When the true population mean $\mu$ is far above the null hypothesis value of 12, there is a low probability that we will make a Type II error.

  Example: $\mu = 14.0$, $\beta = .0104$
Power Curve

Probability of Correctly Rejecting Null Hypothesis vs. \( \mu \)

- \( H_0 \) False

\( \mu \) values range from 11.5 to 14.5.
Determining the Sample Size for a Hypothesis Test About a Population Mean

\[ n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2} \]

where

- \( z_\alpha = z \) value providing an area of \( \alpha \) in the tail
- \( z_\beta = z \) value providing an area of \( \beta \) in the tail
- \( \sigma = \) population standard deviation
- \( \mu_0 = \) value of the population mean in \( H_0 \)
- \( \mu_a = \) value of the population mean used for the Type II error

Note: In a two-tailed hypothesis test, use \( z_{\alpha/2} \) not \( z_\alpha \)
Determining the Sample Size for a Hypothesis Test About a Population Mean

Let’s assume that the director of medical services makes the following statements about the allowable probabilities for the Type I and Type II errors:

• If the mean response time is $\mu = 12$ minutes, I am willing to risk an $\alpha = .05$ probability of rejecting $H_0$.

• If the mean response time is 0.75 minutes over the specification ($\mu = 12.75$), I am willing to risk a $\beta = .10$ probability of not rejecting $H_0$. 
Determining the Sample Size for a Hypothesis Test About a Population Mean

\[ \alpha = .05, \quad \beta = .10 \]
\[ z_\alpha = 1.645, \quad z_\beta = 1.28 \]
\[ \mu_0 = 12, \quad \mu_a = 12.75 \]
\[ \sigma = 3.2 \]

\[ n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2} = \frac{(1.645 + 1.28)^2 (3.2)^2}{(12 - 12.75)^2} = 155.75 \approx 156 \]