Math 450H Fall 2009

Problem Set 6

Approximate the solution to each problem using Euler's, Improved Euler's, and Runga-Kutta methods, and compare the results with the exact solution. Suggested step sizes (h) are given.

Problem 1

$$\begin{aligned} \frac{dy}{dx} &= x + \frac{y}{5}, \quad y(0) = -3, \quad x \in [0, 5] \\ h &= 1, .2 \end{aligned}$$

Problem 2

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{3} y(8 - y), \quad y(0) = 1, \quad x \in [0, 5] \\ h &= 1, .5, .25 \end{aligned}$$

Problem 3

$$\begin{aligned} \frac{dy}{dx} &= y \cos x, \quad y(0) = 1, \quad x \in [0, 20] \\ h &= .4, .2, .1, .05 \end{aligned}$$

For the following problems, implicitly approximate the solution of each using difference quotients and solving the corresponding linear system. Compare the results with the exact solution. Suggested interval numbers (N) are given.

Problem 4

$$y''(t) - 3y'(t) + 2y(t) = 3e^{-t}$$
$$y(0) = 1, \quad y(5) = 4$$
$$N = 5, 10, 20$$

Problem 5

$$y''(t) - 3y'(t) + 2y(t) = 3e^{-t}$$
$$y(0) = 1, \quad y'(5) = 0$$
$$N = 5, 10, 20$$

Problem 6

$$t^2 y''(t) - 2y(t) = 3t^2 - 1$$
$$y(1) = 1, \quad y(5) = 5$$
$$N = 5, 10, 20$$