Math 450H Fall 2009

Problem Set 5

Problem 1 *Consider the following system of equations.*

$$\begin{aligned} \frac{du}{dt} &= -(2+v)(u+v) \\ \frac{dv}{dt} &= -v(1-u) \end{aligned}$$

Qualitatively analyze the long term behavior of the solutions under various initial conditions. In your analysis, include the following:

1. *What are the equilibria?*
2. *Linearize the system about the equilbria.*
3. *Classify the equilibria (spiral sink, saddle, etc.) based on your linearization.*
4. *Classify the stability of the equilibria.*
5. *View a phase plane of the system that shows key trajectories and the equilibria.*

Problem 2 *Consider the following system of equations.*

$$\begin{aligned} \frac{du}{dt} &= v \\ \frac{dv}{dt} &= -4\sin u \end{aligned}$$

Qualitatively analyze the long term behavior of the solutions under various initial conditions. In your analysis, include the considerations mentioned for Problem 1. What physical system does this model?

Problem 3 *Consider the following system of equations.*

$$\begin{aligned} \frac{du}{dt} &= v \\ \frac{dv}{dt} &= -9\sin u - .2v \end{aligned}$$

Qualitatively analyze the long term behavior of the solutions under various initial conditions. In your analysis, include the considerations mentioned for Problem 1. What physical system does this model?

Problem 4 *Consider two dynamic populations (of animals, plants, bacteria, for example) $u(t)$ and $v(t)$ in an enclosed environment. The species u and v compete for the same food source, but otherwise coexist peacefully. Via observation, it is determined that the populations are related by the following system of ordinary differential equations.*

$$\begin{aligned} \frac{du}{dt} &= u(1-u-v) \\ \frac{dv}{dt} &= v(.75-v-.5u) \end{aligned}$$

Qualitatively analyze the long term behavior of the populations under various initial conditions. In your analysis, include the considerations mentioned for Problem 1.

Problem 5 *Consider two dynamic populations $u(t)$ and $v(t)$ in an enclosed environment. This time the species u and v have a predator-prey relationship. Specifically, v is the predator and u is its prey. Via observation, it is determined that the populations are related by the following system of ordinary differential equations.*

$$\begin{aligned} \frac{du}{dt} &= u(1-.5v) \\ \frac{dv}{dt} &= v(-.75+.25u) \end{aligned}$$

Qualitatively analyze the long term behavior of the populations under various initial conditions. In your analysis, include the considerations mentioned for Problem 1.

Problem 6 *Consider the following system of equations.*

$$\begin{aligned} \frac{du}{dt} &= 2u-v+3(u^2-v^2)+2uv \\ \frac{dv}{dt} &= u-3v-3(u^2-v^2)+3uv \end{aligned}$$

Qualitatively analyze the long term behavior of the solutions under various initial conditions. In your analysis, include the considerations mentioned for Problem 1. What physical system does this model?

Problem 7 *Consider the following system of equations.*

$$\begin{aligned} \frac{du}{dt} &= u+v-u\left(u^2+v^2\right) \\ \frac{dv}{dt} &= -u+v-v\left(u^2+v^2\right) \end{aligned}$$

Qualitatively analyze the long term behavior of the solutions under various initial conditions. In your analysis, include the considerations mentioned for Problem 1.

Problem 8 *Consider the following equation.*

$$\frac{d^2x}{dt^2} + \mu\left(x^2 - 1\right)\frac{dx}{dt} + x = 0.$$

Qualitatively analyze solutions for various values of μ. *Suggestions:* $\mu = .2$, $\mu = 1$, $\mu = 5$.