

Math 450H Fall 2009

Problem Set 3

**Problem 1** Review the definitions of algebraic and geometric multiplicities of eigenvalues. How do these multiplicities relate to the possible defects of the following differential equation?

$$\mathbf{x}' = A\mathbf{x} \tag{1}$$

**Problem 2** Develop the equations needed to find a solution to a system with a double defect. Conjecture a generalization to  $n$  defects.

**Problem 3** Solve (1) with  $A = I_{3 \times 3}$  (the identity matrix).

**Problem 4** Solve (1) with  $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$  by first solving for a generalized eigenvalue, then determining the corresponding ordinary eigenvalue (we did the opposite in class).

**Problem 5** Solve (1) with  $A = \begin{bmatrix} 1 & -4 \\ 4 & 9 \end{bmatrix}$ .

**Problem 6** Solve (1) with  $A = \begin{bmatrix} -13 & 40 & -48 \\ -8 & 23 & -24 \\ 0 & 0 & 3 \end{bmatrix}$ .

**Problem 7** Solve (1) with  $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -4 \\ 0 & 1 & -3 \end{bmatrix}$ .

**Problem 8** What is the determinant of each given matrix above? What is the nullspace of each? Given an initial condition  $\mathbf{x}_0$  for each problem, how many solutions should exist to (1)?