Math 450H Fall 2009

Problem Set 2

Problem 1 *Consider the following differential equation.*

$$y''(t) - 3y(t) = 0.$$

Rewrite the differential equation as a 2 dimensional, first order system of differential equations, in matrix-vector form.

Problem 2 *Consider the following two-dimensional system of second-order differential equations.*

$$\begin{aligned} x''(t) + 4x'(t) + 3x(t) - 2y(t) &= 0 \\ y''(t) + 2y'(t) - 1x(t) + 3(t) &= 0 \end{aligned}$$

Rewrite the system as a 4 dimensional system of first order differential equations, in matrix-vector form.

Problem 3 *Consider the 1-dimensional flow of a heat in a uniform rod consisting of 6 cells and 7 faces. Define:*

$$\begin{aligned} u_n(t) &= \textit{temperature in cell } n \textit{ at time } t, \textit{ for } n = 0, \ldots, 7 \\ \Delta_m(t) &= \textit{difference in temperature across face } m \textit{ at time } t, \textit{ for } m = 1, \ldots, 7 \\ \phi_m(t) &= \textit{heat flow across face } m \textit{ at time } t \\ k_m &= \textit{thermal conductivity of face } m \end{aligned}$$

Suppose the heat in "cells" 0 and 7 are both zero. Given that the flow proportionality constants are all equal to one, and

$$k_1 = 1, \quad k_2 = 1, \quad k_3 = 4, \quad k_4 = 1, \quad k_5 = 4, \quad k_6 = 1, \quad k_7 = 3$$

1. *Write the resulting system of ODEs in the matrix-vector form*

   $$\mathbf{u}'(t) = A\mathbf{u}(t),$$

   specifying the matrix A.

2. *Assume the initial temperature vector is*

$$\mathbf{u}(0)^T = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

   and solve for $u_n(t)$, $n = 1, \ldots, 6$. *Use the orthogonality of eigenvectors to specify the vector solution.*

3. *Plot the solutions on a single plot over an appropriate domain.*

Problem 4 *Change the initial vector in the previous problem to look at several different conditions and resulting heat flow. What happens if all intial temperature are 1) zero 2) equal?*