

MATH 305
Linear Systems of ODEs

Problem 1 Consider the system of equations

$$\begin{aligned}x'(t) &= -y(t) \\ y'(t) &= 13x(t) + 4y(t).\end{aligned}$$

1. Rewrite this system in matrix vector form. More specifically, construct the matrix A :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}$$

2. Rewrite the system as a single, 2nd order ODE.

Problem 2 Consider the following 3rd order ODE.

$$t^3 x^{(3)}(t) - 2t^2 x''(t) + 3tx'(t) + 5x(t) = \ln t.$$

1. Define $x_1 = x$, $x_2 = x'_1$, and $x_3 = x'_2$. Rewrite the 3rd order ODE in first order form using substitution.

2. Rewrite the 3rd order ODE as a system of 3, 1st order ODEs.

3. Write the result of part 2 in matrix vector form.

Problem 3 Consider the two 2nd order ODEs given below.

$$\begin{aligned}x''(t) + y'(t) + x(t) &= t - 3 \\y''(t) + x'(t) + x(t) - y(t) &= t^2\end{aligned}$$

Rewrite this two dimensional, second order system as a four dimensional, first order system.