MATH 305
Lab 1, Part 3
Direction field homework

For the differential equations in Problems 1-4, print out the associated direction fields with a display window of approximately $t \in [-5, 5]$ and $y \in [-5, 5]$. Show the solution curves through the initial points $(t_0, y_0) = (0, 0), (-2, -1), (-3, 2), (0, 1)$, and $\left(4, -\frac{1}{2}\right)$. Print out the display window and turn it in as part of this assignment.

Problem 1 $y' = ty$

Problem 2 $y' = y^2 - t^2$

Problem 3 $y' = \dfrac{2ty}{1 + y^2}$

Problem 4 $y' = y(2 + y)(2 - y)$

For the differential equations in Problems 5-8, perform the following tasks.

a) Plot the associated direction fields, along with a few solutions with different initial points. Start with the display window bounded by $0 \le t \le 10$ and $-5 \le y \le 5$, and modify it to suit the problem. Print out the display window and turn it in as part of this assignment.

b) Make a conjecture about the limiting behavior of the solutions as $t \to \infty$.

Problem 5 $y' + 3y = 8$

Problem 6 $(1 + t^2)\, y' + 5ty = t$

Problem 7 $ty' + ty = 15 - y$

Problem 8 $(1 + t)\, y' = y(4 - y^2)$

Problems 9-10 are on the second page.

For Problems 9-10, consider a certain lake which has a volume of $V = 100km^3$. It is fed by a river at a rate of r_i $km^3/year$, and there is another river which is fed by the lake at a rate which keeps the volume of the lake constant. In addition, there is a factory on the lake which introduces a pollutant into the lake at the rate of p $km^3/year$. This means that the rate of flow from the lake into the outlet river is $(p + r_i)$ $km^3/year$. Let $x(t)$ denote the volume of the pollutant in the lake at time t, and let $c(t) = x(t)/V$ denote the concentration of the pollution. Under the assumption of immediate and perfect mixing of the pollutant into the lake water, the concentration satifies the differential equation

$$\frac{dc}{dt} + \left(\frac{p + r_i}{V}\right) c = \frac{p}{V}$$

Problem 9 *Suppose that* $r_i = 50$ *and* $p = 2$.

1. *Assume that the factory starts operating at time* $t = 0$*, so that the initial concentration is* 0*. Use a direction field to plot the associated solution. Remember the definition of the concentration is* x/V*, so you can be sure it is pretty small. Choose the dimensions of the display window carefully.*

2. *It has been determined that a concentration of over* 2.5% *is hazardous for the fish in the lake. Approximately how long will it take until this concentration is reached?*

3. *What is the limiting concentration? About how long does it take for the concentration to reach approximately 3.6%?*

Problem 10 *Suppose the factory stops operating at time* $t = 0$*, and that the concentration was* 3.6% *at that time. Approximately how long will it take before the concentration falls below* 2.5%*, and the lake is no longer hazardous for fish? Notice that* $p = 0$ *for this problem.*