Math 305 Spring 2011

Assignment 5

The due date for this assignment is Wednesday, March 9.

Instructions: You are encouraged to use a computer to aid with otherwise tedious computations and plotting. In these cases, provide a printout for full credit. Additionally, be sure to demonstrate your knowledge of and competence with the techniques discussed in class, for full credit.

Problem 1 *Section 4.6, #2*

Problem 2 *Section 4.6, #8*

Problem 3 *Section 4.6, #12*

Problem 4 *Solve the following differential equation.*

$$y''(t) + 4y'(t) + 4y(t) = t^{-2}e^{-2t}, \qquad t > 0$$

*For the IVPs in Problems 5 and 6, use the representation $y_p(t) = \int_0^t F(t,s)\, f(s)\, ds$, where $F(t,s)$ is defined as in class lecture.

Problem 5 *Use variation of parameters* to determine the solution of*

$$y''(t) + y(t) = \begin{cases} t, & 0 \le t \le \pi \\ \pi e^{\pi - t}, & t > \pi \end{cases},$$

satisfying the initial conditions $y(0) = 0$ and $y'(0) = 1$. Plot the solution on a reasonable time domain.

Problem 6 *Consider the IVP:*

$$y''(t) + \omega^2 y(t) = f(t), \qquad y(0) = A, \qquad y'(0) = B.$$

Use variation of parameters to solve if*

$$f(t) = \begin{cases} t-2, & 0 \le t < 1 \\ t, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}.$$

Problem 7 *Consider a forced, undamped spring-mass system modeled by*

$$x''(t) + 16x(t) = 10\cos(\omega t), \qquad \omega \neq \pm 4. \tag{1}$$

1. *Solve (1).*

2. *Given that*

$$x(0) = x'(0) = 0, \tag{2}$$

construct three separate plots of solutions for the cases $\omega = 3$, $\omega = 3.5$, and $\omega = 3.9$. Plot $[0, T]$, where $T \geq 2$ periods.

Problem 8 *Solve the initial value problem given by equation (1) and conditions (2), for the case $\omega = 4$. Plot your solution on a reasonable domain.*