

## Math 305 Spring 2011

### Assignment 4

The due date for this assignment is Wednesday, March 2.

**Instructions:** Feel free to use a computer to aid with otherwise tedious computations and plotting. In these cases, provide a printout for full credit. Additionally, be sure to demonstrate your knowledge of and competence with the techniques discussed in class, for full credit.

**Problem 1** Consider an unforced spring-mass-dashpot model, with

$$m = 2, \quad c = 20, \quad k = 250, \quad x(0) = 5, \quad v(0) = 40.$$

Note that  $x(t)$  and  $v(t)$  represent the position and velocity functions, respectively. Also,  $m$  is the mass attached to both a spring (with spring constant  $k$ ) and a dashpot (with damping constant  $c$ ).

1. Determine the position function  $x(t)$  and whether the motion is overdamped, critically damped, or underdamped.
2. Plot the solution on the domain  $t \in [0, 1]$ .

**Problem 2** Consider the initial value problem

$$\begin{aligned} x''(t) + px'(t) + 9x(t) &= 0, \quad t \geq 0, \\ x(0) = 1 \quad \text{and} \quad x'(0) &= 1. \end{aligned}$$

1. Find the positive value  $P_0$  such that the solution to the differential equation oscillates when  $0 \leq p < P_0$ , but no longer oscillates if  $p \geq P_0$ .
2. Given that  $p$  causes oscillations, solve the IVP. Do not specify a value for  $p$  – solve in the general case.

**Problem 3** Section 4.5, #8

**Problem 4** Section 4.5, #16

**Problem 5** Section 4.5, #28

**Problem 6** *Section 4.5, #38*

**Problem 7** *Section 4.5, #40*

For Problems 8, 9, and 10, solve the differential equations using the method of undetermined coefficients.

**Problem 8**  $y''(t) + 2y'(t) - 3y(t) = 1 + te^t$

**Problem 9** *Let  $\lambda > 0$  and  $\lambda \neq n\pi$ .*

$$y''(t) + \lambda^2 y(t) = \sum_{n=1}^N a_n \cos(n\pi t)$$

**Problem 10**  $y''(t) - 6y'(t) + 13y(t) = te^{3t} \sin(3t)$