

MATH 105
Final exam review

Problem 1 Determine the following limit.

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}.$$

Problem 2 Determine the following limit and find horizontal asymptotes (if any).

$$\lim_{x \rightarrow +\infty} \frac{-x^3 + 2x + 1}{x - 3}.$$

Problem 3 Determine if the function

$$f(x) = \begin{cases} 2x^2 - x, & x < 3 \\ 3 - x, & x \geq 3 \end{cases}$$

is continuous at $x = 3$

Problem 4 Use the limit definition of the derivative to show that

$$\frac{d}{dx} [x^2] = 2x.$$

Problem 5 Given that the position at time t of an object moving along a line is

$$s(t) = t^3 - 6t^2 + 9t + 5,$$

find the velocity (the rate of change of position with respect to time) of the object at $t = 4$.

Problem 6 Given $y(x) = x^2(3x + 1)$, find $y''(x)$.

Problem 7 Find the second derivative (with respect to x) of $f(x) = \frac{3x - 2}{(x - 1)^2}$.

Problem 8 Find $\frac{dy}{dx}$ if $x^2y + y^2 = x^3$.

Problem 9 Find the intervals of increase and decrease and the relative extrema of the function $f(t) = \sqrt{3 - 2t - t^2}$.

Problem 10 Determine the locations of points of inflection and intervals of concavity for the function $f(x) = 2x^6 - 5x^4 + 7x + 3$.

Problem 11 For several weeks, the highway department has been recording the speed of freeway traffic flowing past a certain downtown exit. The data suggest that between 1:00 and 6:00 p.m. on a normal weekday, the speed of the traffic at the exit is approximately $S(t) = t^3 - 10.5t^2 + 30t + 20$ miles per hour, where t is the number of hours past noon. At what time between 1:00 and 6:00 p.m. is the traffic moving the fastest, and at what time is it moving the slowest?

Problem 12 Given $f(x)$, determine $\frac{df}{dx}$.

1. $f(x) = \ln(2x^3 + 1)$

2. $f(x) = e^{x^2+1}$

Problem 13 Compute the following integral:

$$\int \left(\frac{x^3 + 2x - 7}{x} \right) dx.$$

Problem 14 Compute the following integral:

$$\int \frac{(\ln x)^2}{x} dx.$$

Problem 15 A protein with mass m (grams) disintegrates into amino acids at a rate given by

$$\frac{dm}{dt} = \frac{-30}{(t+3)^2} \text{ grams/hour.}$$

Determine the net change in mass of the protein during the first two hours

Problem 16 Find the area of the region enclosed by the curves $y = x^3$ and $y = x^2$.

Problem 17 Compute the following limit.

$$\lim_{x \rightarrow 1} \frac{x^5 - 3x^4 + 5x - 3}{4x^5 + 2x^3 - 5x^2 - 1}$$