Hypothesis Tests

Chapter 9

Hypotheses

- Hypothesis – a theory, assumption, supposition based on limited information. An unproven statement.
- On average, a 20 ounce bottle of *So Duh!* contains 20 ounces of the soft drink.
- Q’s new order-filling process will reduce the average customer wait time.
- 30% of triathletes ran cross country in high school.
- More than ¾ of all dentists prefer *Brand X* to *Brand Y.*

Hypothesis Tests

- Method to determine whether or not to reject the hypothesis

Hypothesis Tests

- The **null hypothesis** is denoted H_0
  - the assumption to be challenged

- The **alternative hypothesis** is denoted H_a. It is the complement of H_0.
  - the thing that must be "proved" or what we need to be convinced of
  - the research hypothesis

Hypothesis Tests

- Statement of the null hypothesis
- Statement of the alternative hypothesis
- Examples

$H_0: \mu = \mu_0 \quad H_a: \mu \neq \mu_0$

$H_0: \mu \leq \mu_0 \quad H_a: \mu > \mu_0$

$H_0: \mu \geq \mu_0 \quad H_a: \mu < \mu_0$

Hypothesis Tests

- Q’s new order-filling process will reduce the average customer wait time…. Is this process faster?

  null hypothesis?

- Is the average amount of soda in a bottle 20 ounces?

  null hypothesis?

Hypothesis Tests

- Note: perspective can drive the choice of the null hypothesis.
- Example: On average, a 20 ounce bottle of *So, Duh!* contains 20 ounces of the soft drink.
- Company perspective:

  $H_0: \mu = 20 \quad H_a: \mu \neq 20$
- Consumer protection group perspective:

  $H_0: \mu \geq 20 \quad H_a: \mu < 20$

Errors

- Reject the null hypothesis when it should have been accepted, or accept the null hypothesis when the alternative hypothesis is true.

Reality

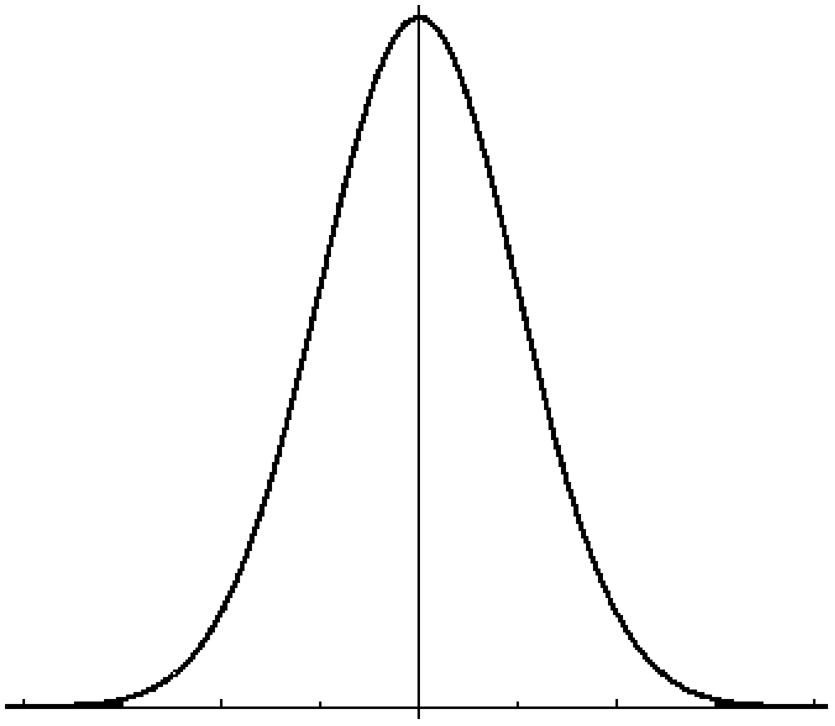
Conclusion		**H_0 is True**	**H_a is True**
	Accept H_0	Correct conclusion	Type II Error
	Reject H_0	Type I error	Correct conclusion

Hypothesis Tests

- The **level of significance**, α, is the probability of making a Type I error.
- If the probability of making a Type II error has not been determined or controlled, say "do not reject H_0" instead of "accept H_0."

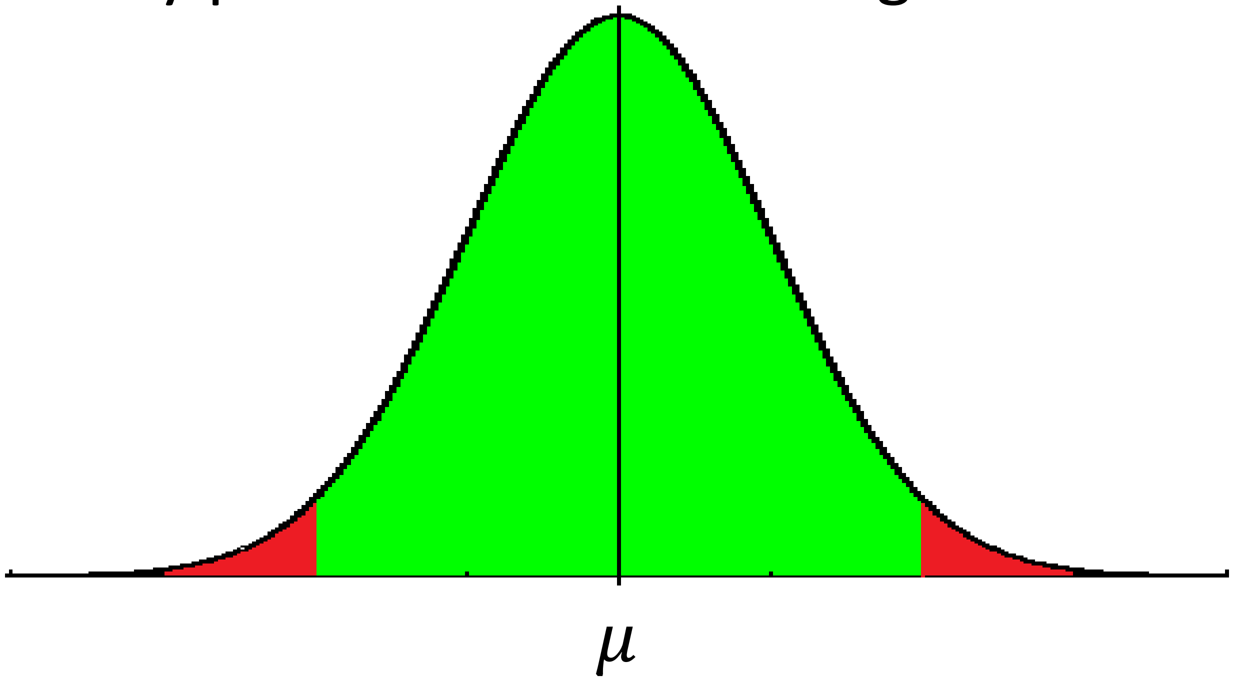
Hypothesis Tests: Population Mean, σ Known

- Recall – the sampling distribution of the sample mean is normally distributed about the population mean if the data is normally distributed, or if the sample size is large enough
- $\mu_{\bar{x}} = \mu$
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$



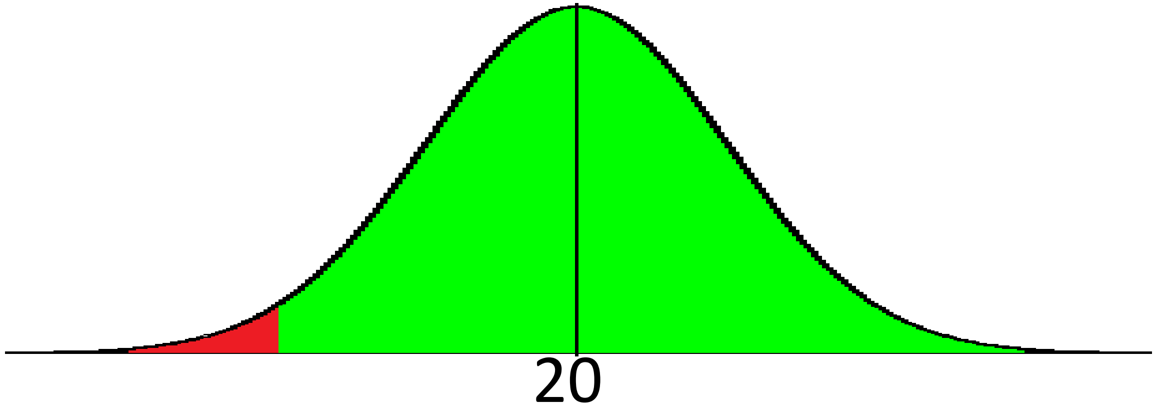
Hypothesis Tests: Population Mean, σ Known

- So, if we know the population standard deviation, we can use the standard normal curve to compute how likely it is that we obtain a sample mean of any particular value or range of values.



Hypothesis Tests: Population Mean, σ Known

- Example: *So, Duh!* problem, consumer protection perspective
- H_0: $\mu \geq 20$
- H_a: $\mu < 20$



- One-Tailed Test

$$H_0:\ \mu \geq \mu_0;\ H_a:\ \mu < \mu_0 \text{ (lower tail test)}$$

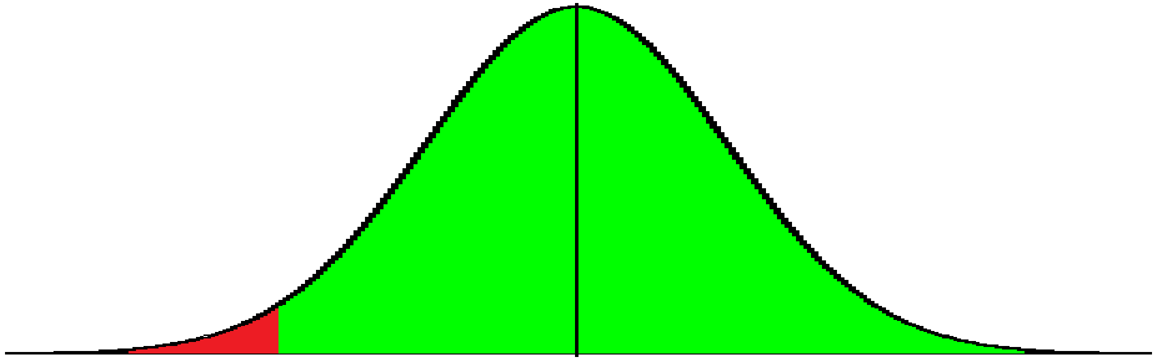
$$H_0:\ \mu \leq \mu_0;\ H_a:\ \mu > \mu_0 \text{ (upper tail test)}$$

So Duh! problem, consumer protection perspective

- Assume that the population standard deviation is known to be 0.5 ounces.
- We only want to accuse the company of under-filling their bottles if we're "really sure" that they are.
- How sure?

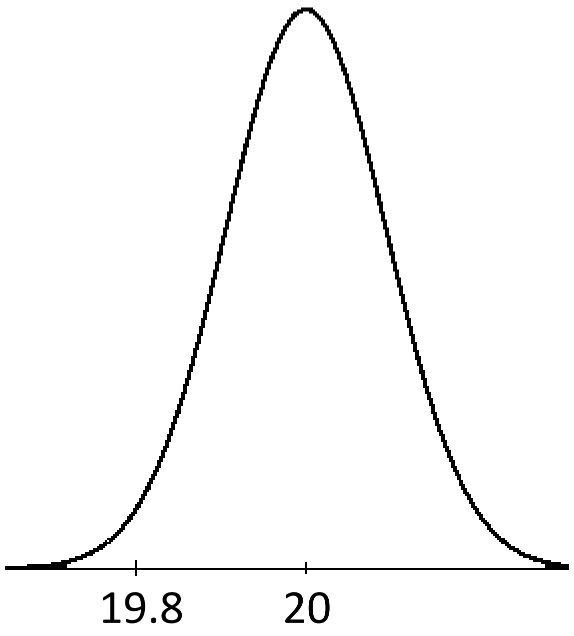
  Let's go with 99% certainty

- Thus, we're willing to risk a 1% chance of a false accusation (rejecting the null hypothesis when it's true – a type I error). This is our level of significance.



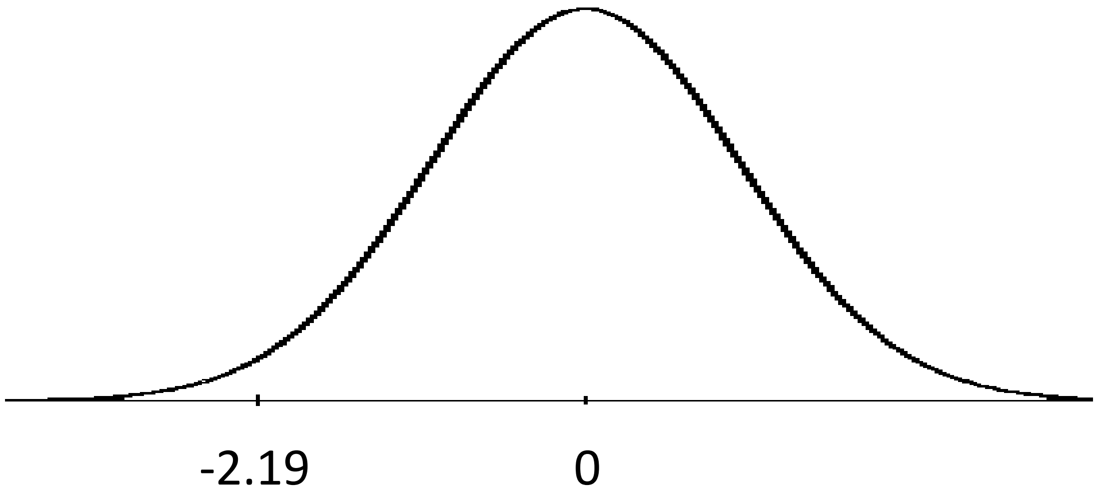
So Duh! problem, consumer protection perspective

- Assume we create a random sample 30 bottles of soda and find that the sample mean volume is 19.8 ounces.
- Do we think they're under-filling their bottles?
- How sure are we?
- $\mu_{\bar{x}} = \mu = 20$
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{30}} \approx 0.091287$



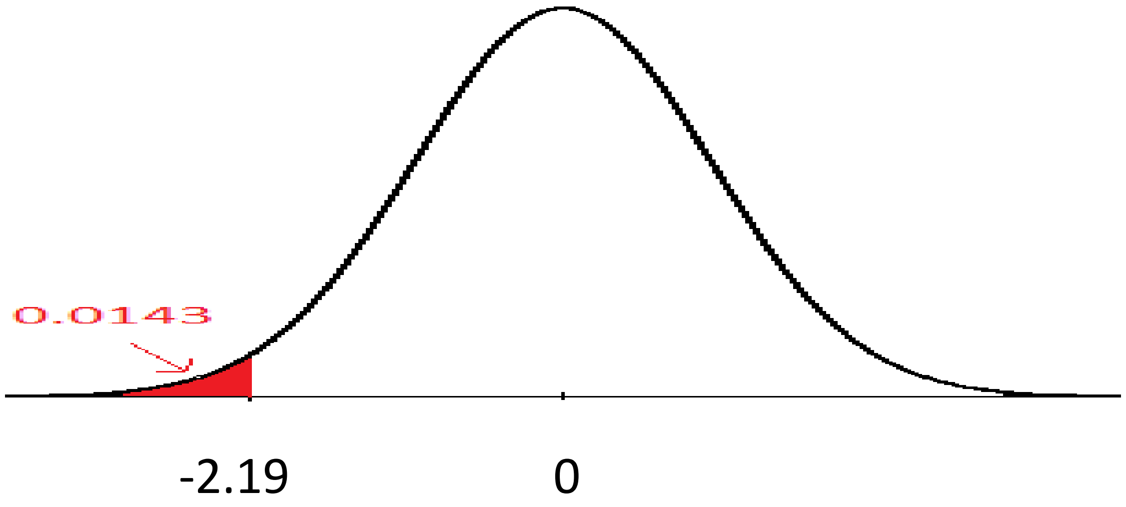
So Duh! problem, consumer protection perspective

- $\mu_{\bar{x}} = \mu = 20$
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{30}} \approx 0.091287$
- Compute the corresponding z for 19.8
- $z = \frac{19.8-20}{0.091287} = -2.19089$
- This z is called our <u>**test statistic**</u>



So Duh! problem, consumer protection perspective

- $\mu_{\bar{x}} = \mu = 20$
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{30}} \approx 0.091287$
- Compute the corresponding z for 19.8
- $z = \frac{19.8 - 20}{0.091287} = -2.19089$
- This z is called our **test statistic**
- The area under the standard normal curve to the left of this test statistic is 0.0143
- This is called our **p-value**
- Since 0.0143 is not less than or equal to $\alpha = 0.01$, we cannot reject the null hypothesis at the 1% significance level



So Duh! problem, consumer protection perspective

- We call the approach we just used to determine whether to reject H0 the p-value approach.
- Equivalently, we could have compared the test statistic to the critical value (z_{α}, in this case approximately -2.33). This method is called the critical value approach.

Summary of One-Tailed Tests (σ Known)

- Lower Tail Test

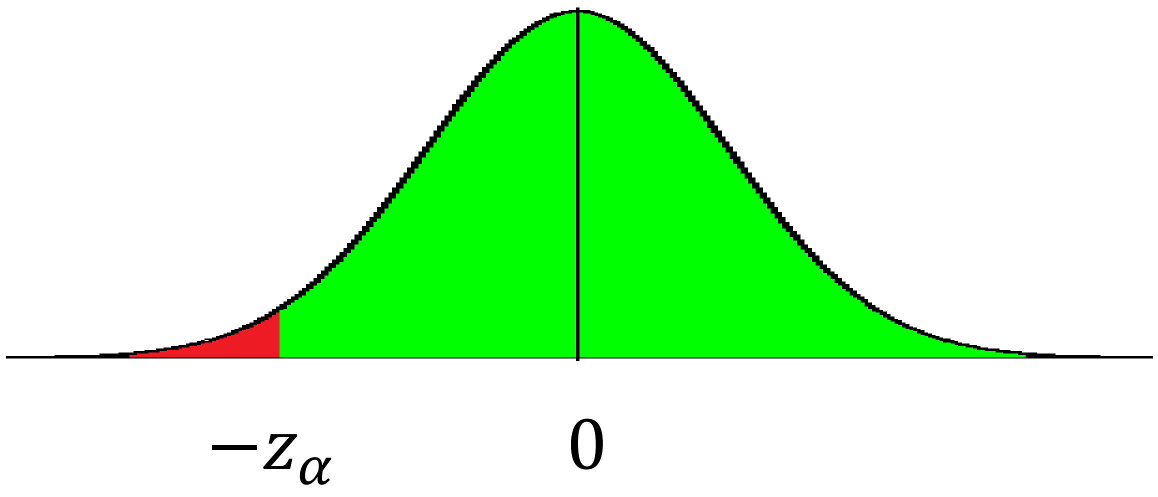
State the hypotheses: H_0: $\mu \geq \mu_0$; H_a : $\mu < \mu_0$

Compute the test statistic: $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$

Reject H_0 if: p-value $\leq \alpha$

equivalently

Reject H_0 if: $z \leq -z_\alpha$



Summary of One-Tailed Tests (σ Known)

- Upper Tail Test

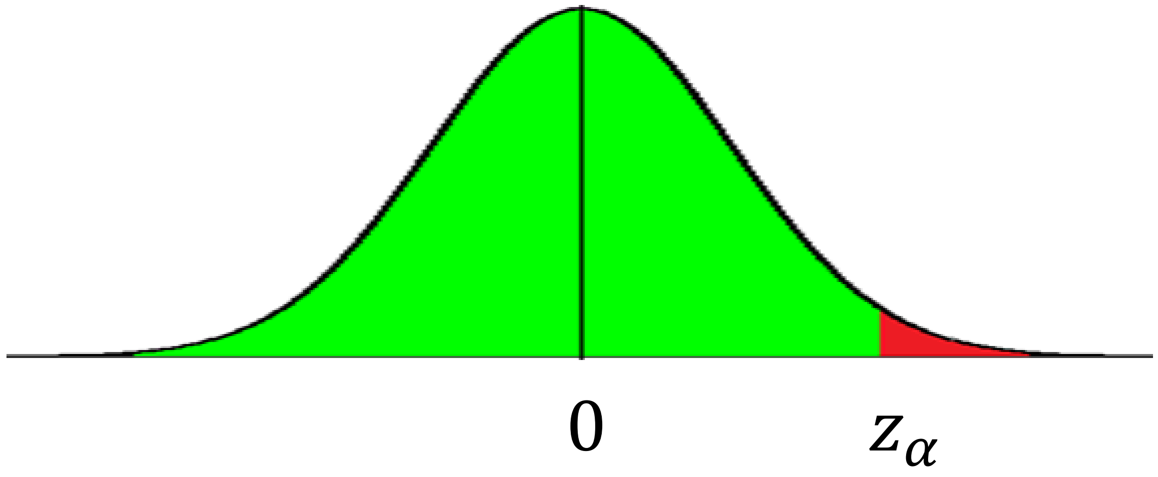
State the hypotheses: H_0: $\mu \leq \mu_0$; H_a : $\mu > \mu_0$

Compute the test statistic: $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$

Reject H_0 if: p-value $\leq \alpha$

equivalently

Reject H_0 if: $\mathrm{z} \geq z_\alpha$



Problem 16 on page 369 of your textbook

(Anderson, Sweeny, Williams. *Statistics for Business and Economics,* 11e)

In a study entitled How Undergraduate Students Use Credit Cards, it was reported that undergraduate students have a mean credit card balance of \$3173 (*Sallie Mae,* April 2009). This figure was an all-time high and had increased 44% over the previous five years. Assume that a current study is being conducted to determine if it can be concluded that the mean credit card balance for undergraduate students has continued to increase compared to the April 2009 report. Based on previous studies, use a population standard deviation of \$1000.

a. State the null and alternative hypotheses.
b. What is the p-value for a sample of 180 undergraduate students with a sample mean credit card balance of \$3325?
c. Using a 0.05 level of significance, what is your conclusion?