Math 370

Section 7.4: Linear Programming III: The Simplex Method

The Simplex Method, developed by George Dantzig, incorporates both optimality and feasibility tests to find the optimal solution(s) to a linear porgram (if one exists).

An optimality test shows whether or not an intersection point corresponds to a value of the objective function better than the best value found so far.

A feasibility test determines whether the proposed intersection point is feasible.

To implement the Simplex Method, we must separate the decision and slack variables into two disjoint sets: independent (initially, the decision variables) and dependent (initially, the slack variables).

Steps of the Simplex Method

1. **Tableau Format**

Adjoin slack variables as needed to convert inequality constraints to equalities. Remember that all variables are nonnegative. Include the objective function constraint as the last constraint, including its slack variable z.

2. **Initial Extreme Point**

The Simplex Method begins with a known extreme point, usually the origin (0,0).

3. **Optimality Test**

Determine whether an adjacent intersection point improves the value of the objective function. If not, the current extreme point is optimal. If an improvement is possible, the optimality test determines which variable currently in the independent set (having value zero) should enter the dependent set and become nonzero.

To do this, examine the last equation (which corresponds to the objective function). If all its coefficients are nonnegative, then stop: the current extreme point is optimal. Otherwise, some variables have negative coefficients, so choose the variable with the largest (in absolute value) negative coefficient as the new entering variable.

4. **Feasibility Test**

To find a new intersection point, one of the variables in the dependent set must exit to allow the entering variable from Step 3 to become dependent. The feasibility test determines which current dependent variable to choose for exiting, ensuring feasibility.

To do this, divide the current right-hand side values by the corresponding coefficient values of the entering variable in each equation. Choose the exiting variable to be the one corresponding to the smallest positive ratio after this division.

5. **Pivot**

Form a new equivalent system of equations by eliminating the new dependent variable from the equations that do not contain the variable that exited in Step 4. Then set the new independent variables to zero in the new system to find the values of the new dependent variables, thereby determining an intersection point.

Repeat Steps 3-5 until an optimal extreme point is found.