Math 370

Linear Programming I: Geometric Solutions

A linear program has the important property that the points satisfying the constraints form a convex set:

A set U in a vector space is called convex if for any $x, y \in U$ and $\alpha \in (0, 1)$ we have $\alpha x + (1 - \alpha)y \in U$. That is, a set is convex if for every pair of points in the set, the line segment joining them lies wholly in the set.

An extreme point of a convex set is any boundary point in the convex set that is the unique intersection point of two of the boundary segments.

Theorem: Suppose the feasible region of a linear program is a nonempty, closed, and bounded convex set. Then the objective function must attain both a maximum and a minimum value occurring at the extreme points of the region.

Note: If the feasible region is unbounded and/or not closed, the objective function need not assume its optimal values. If either a maximum or minimum does exist, it must occur at one of the extreme points.

Example: Carving Wooden Soldiers

Consider a company that carves wooden soldiers. The company specializes in two main types: Confederate and Union soldiers. The profit for each is \$28 and \$30, respectively. It requires 2 units of lumber, 4 hours of carpentry, and 2 hours of finishing to complete a Confederate soldier. It requires 3 units of lumber, 3.5 hours of carpentry, and 3 hours of finishing to complete a Union soldier. Each week the company has 100 units of lumber delivered. There are 120 hours of carpenter machine time available and 90 hours of finishing time available. Determine the number of each wooden soldier to produce to maximize weekly profits.