Math 370

An Overview of Optimization Modeling

The basic optimization model is

$$\text{Optimize } f_k(X) \text{ for } k \text{ in } K$$

subject to

$$g_i(X) \left\{ \begin{array}{c} \geq \\ = \\ \leq \end{array} \right\} b_i \text{ for all } i \text{ in } I.$$

f_k are the objective functions, X is the decision variable, and the relationship between g_i and b_i are the constraint equations.

An optimization problem is said to be unconstrained if there are no constraints and constrained if one or more side conditions are present.

An optimization problem is said to be a linear program if it satisfies the following properties:

1. There is a unique objective function.
2. Whenever a decision variable appears in either the objective function or one of the constraint functions, it must appear only as a power term with an exponent of 1, possibly multiplied by a constant.
3. No term in the objective function or in any of the constraints can contain products of the decision variables.
4. The coefficients of the decision variables in the objective function and each constraint are constant.
5. The decision variables are permitted to assume fractional as well as integer values.

Problems with more than one objective function are called multiobjective or goal programs.

Optimization problems that fail Properties 2 and/or 3 (listed above) are said to be nonlinear.

If the coefficients are time dependent, the resulting problem is called a dynamic program.

If the coefficients are probabilistic, the resulting problem is called a stochastic program.

If one or more of the decision variables are restricted to integer values, the resulting problem is called an integer program.

Example 1: Determining a Production Schedule

A carpenter makes tables and bookcases. He is trying to determine how many of each type of furniture he should make each week. The carpenter wishes to determine a weekly production schedule for tables (t) and bookcases (b) that maximizes his profits. It costs \$5 and \$7 to produce tables and bookcases, respectively. The revenues are estimated by the expressions

$$50t - 0.2t^2$$

and

$$65b - 0.3b^2.$$

Example 2: Determining a Production Schedule Revisited

The carpenter realizes a net unit profit of \$25 per table and \$30 per bookcase. He is trying to determine how many of each piece of furniture he should make each week. He has up to 690 board-feet of lumber to devote weekly to the project and up to 120 hr of labor. He can use lumber and labor productively elsewhere if they are not used in the production of tables and bookcases. He estimates that it requires 20 board-feet of lumber and 5 hr of labor to complete a table and 30 board-feet of lumber and 4 hr of labor for a bookcase. Moreover, he has signed contracts to deliver four tables and two bookcases every week. The carpenter wishes to determine a weekly production schedule for tables and bookcases that maximizes his profits.

Example 3: Space Shuttle Cargo

There are various items to be taken on a space shuttle. Unfortunately, there are restrictions on the allowable weight and volume capacities. Suppose there are m different items, each given some numerical value c_j and having weight w_j and volume v_j. The goal is to maximize the value of the items that are to be taken without exceeding the weight limitation W or the volume limitation V.

Example 4: An Investment Problem

An investor has \$40,000 to invest. She is considering investments in savings at 7%, municipal bonds at 9%, and stocks that have been consistently averaging 14%. Because there are varying degrees of risk involved in the various investments, the investor has listed the following goals for her portfolio:

1. A yearly return of at least \$5000.
2. An investment of at least \$10,000 in stocks.
3. The investment in stocks should not exceed the combined total in bonds and savings.
4. A liquid savings account between \$5000 and \$15,000.
5. The total investment must not exceed \$40,000.