**Math 300**

**Section 5.5** Complex Eigenvalues

A complex number is of the form $z = a + bi$, where $a$ and $b$ are real numbers and $i = \sqrt{-1}$. The real part of $z$ is $Re\ z = a$ and the imaginary part of $z$ is $Im\ z = b$.

The conjugate of $z = a + bi$ is $\bar{z} = a - bi$. The absolute value of $z = a + bi$ is $\sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$.

The angle between the positive real axis and the line from the origin to $z$ is the Argument of $z$, which is denoted $\phi = Arg\ z$. Notice that if $z = a + bi$, then $a = |z| \cos\phi$ and $b = |z| \sin\phi$. The complex number $z$ can be expressed in polar form:

$$z = |z|(\cos\phi + i \sin\phi).$$

If the matrix $A$ has a complex eigenvalue $\lambda$ with corresponding eigenvector $\mathbf{x}$, then $\bar{\lambda}$ is also an eigenvalue of $A$ with corresponding eigenvector $\bar{\mathbf{x}}$.

**Theorem** Let $A$ be a real $2 \times 2$ matrix with a complex eigenvalue $\lambda = a - bi$ and an associated eigenvector $\mathbf{v}$ in $\mathbb{C}^2$. Then $A = PCP^{-1}$, where $P = [Re\ \mathbf{v} \ \ Im\ \mathbf{v}]$ and $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$.