Math 300

Section 2.8 Subspaces of $\mathbb{R}^n$

A subspace of $\mathbb{R}^n$ is any set H in $\mathbb{R}^n$ that has three properties:

1. The zero vector is in H.
2. For each $\mathbf{u}$ and $\mathbf{v}$ in H, $\mathbf{u} + \mathbf{v}$ is in H.
3. For each $\mathbf{u}$ in H and each scalar c, $c\mathbf{u}$ is in H.

The column space of a matrix A is the set $ColA$ of all linear combinations of the columns of A.

The null space of a matrix A is the set $NulA$ of all solutions of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

Theorem If A is an $m \times n$ matrix, $ColA$ is a subspace of $\mathbb{R}^m$ and $NulA$ is a subspace of $\mathbb{R}^n$.

A basis for a subspace H of $\mathbb{R}^n$ is a linearly independent set in H that spans H.

Theorem The pivot columns of a matrix A form a basis for the column space of A.