Math 300

Section 2.7 Applications to Computer Graphics

Consider the problem of plotting a simple picture in 2-D consisting only of points connected by lines. To do this, you need to know two things: the points, and which points are connected to which others. We keep track of the points by putting them in the columns of a matrix called a data matrix. A way to record which points connect to which others is to use an adjacency matrix. This matrix consists only of 0's and 1's; the (i, j) entry in the matrix is a 1 if points i and j are connected.

Dilations and Contractions

A dilation transformation has standard matrix $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, which stretches the x-axis by a factor of a and the y-axis by a factor of b.

Shears

A horizontal shear transformation has standard matrix $B = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$, which turns vertical lines into lines of slope k.

Rotations

A counterclockwise rotation by ϕ radians has the standard matrix $B = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$.

Homogeneous Coordinates

A major problem comes when we want to translate the figure; that is, just move it sideways, up and down, or diagonally. In math terms we want to move the point (x, y) to the point $(x + h, y + k)$ for some constants h and k. It turns out that these transformations are not linear. To solve this problem we use homogeneous coordinates: we identify the point (x, y) in 2-space with the point $(x, y, 1)$ in 3-space, and look for a transformation that maps $(x, y, 1)$ to $(x + h, y + k, 1)$. This transformation is linear and has standard matrix $B = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}$.