Math 300

Section 2.2 The Inverse of a Matrix

Let A be an $n \times n$ matrix. If there is a matrix C with $AC = CA = I_n$, then A is invertible and C is the inverse of A. This is denoted $C = A^{-1}$.

Theorem If A is an invertible $n \times n$ matrix, then for each $\mathbf{b} \in \mathbb{R}^n$, the matrix equation $A\mathbf{x} = \mathbf{b}$ has the unique solution $\mathbf{x} = A^{-1}\mathbf{b}$.

Properties of the Inverse

1. If A is invertible, then A^{-1} is invertible and $(A^{-1})^{-1} = A$.
2. If A and B are invertible, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.
3. If A is invertible, then A^T is invertible and $(A^T)^{-1} = (A^{-1})^T$.

When is a matrix invertible?

An $n \times n$ matrix is invertible if and only if it is row equivalent to I_n.

Elementary Matrices

An elementary matrix results from performing a single row operation on I_n.

An Algorithm for finding A^{-1}

Row reduce the augmented matrix $[A|I]$. If A is row equivalent to I, then $[A|I]$ is row equivalent to $[I|A^{-1}]$. Otherwise, A does not have an inverse.