Math 300

Section 2.1 Matrix Operations

The number a_{ij} is the (i, j) entry of a matrix A. The (i, j) entry of A lies in the ith row and jth column of A.

The zero matrix 0 is the $m \times n$ matrix whose entries are all zeroes.

Two matrices are equal if they have the same size and if their corresponding entries are equal.

Addition

The sum $A + B$ of two $m \times n$ matrices is an $m \times n$ matrix whose entries are the sum of the corresponding entries in A and B.

Scalar Multiplication

If r is a scalar and A is a matrix, then the scalar multiple rA is the matrix whose entries are the corresponding entries of A multiplied by r.

Properties of Addition and Scalar Multiplication

Let A, B, and C be matrices of the same size, and let r and s be scalars. Then

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + 0 = A$
4. $r(A + B) = rA + rB$
5. $(r + s)A = rA + sA$
6. $r(sA) = (rs)A$

Multiplication

$$AB = [A\mathbf{b}_1 \ A\mathbf{b}_2 \ \cdots \ A\mathbf{b}_p]$$

Properties of Matrix Multiplication

Let A be an $m \times n$ matrix, let B and C be matrices of appropriate size, and let r be a scalar. Then

1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$
3. $(B + C)A = BA + CA$
4. $r(AB) = (rA)B = A(rB)$
5. $I_m A = A = AI_n$ where I_n is the $n \times n$ matrix with ones along the diagonal and zeroes elsewhere

Warnings

1. In general, $AB \neq BA$, even when AB and BA are both defined and are the same size. If $AB = BA$, we say that A and B commute.
2. It is not true in general that $AB = AC$ implies that $B = C$.
3. If is not true in general that $AB = 0$ implies that $A = 0$ or $B = 0$.

Powers of a Matrix

If A is an $n \times n$ matrix, then one can use matrix multiplication to define positive integer powers of A :

$$A^k = A\dot{A} \cdots A (k \text{ times })$$

(Note: $A^0 = I_n$.)

Transpose

The transpose A^T of an $m \times n$ matrix is an $n \times m$ matrix whose ith column is the ith row of A.

Properties of the Transpose

Let A be an $m \times n$ matrix, let B be a matrix of appropriate size, and let r be a scalar. Then

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$
3. $(rA)^T = rA^T$
4. $(AB)^T = B^T A^T$