Math 300

Section 1.8 Introduction to Linear Transformations

A transformation (or function or mapping) T from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a rule that assigns to each vector $\mathbf{x}$ in $\mathbb{R}^n$ a vector $T(\mathbf{x})$ in $\mathbb{R}^m$. The set $\mathbb{R}^n$ is called the domain of T, and $\mathbb{R}^m$ is called the codomain of T. For $\mathbf{x}$ in $\mathbb{R}^n$, the vector $T(\mathbf{x})$ in $\mathbb{R}^m$ is called the image of $\mathbf{x}$ and the set of all images $T(\mathbf{x})$ is called the range of T. *Notation*: $T : \mathbb{R}^n \to \mathbb{R}^m$.

A transformation (or mapping) T is linear if:

1. $T(\mathbf{u} + \mathbf{v}) = T\mathbf{u} + T\mathbf{v}$ for all $\mathbf{u}, \mathbf{v}$ in the domain of T.
2. $T(c\mathbf{u}) = cT(\mathbf{u})$ for all scalars c and all $\mathbf{u}$ in the domain of T.

Theorem T is a linear transformation if and only if

$$T(c\mathbf{u} + d\mathbf{v}) = cT(\mathbf{u}) + dT(\mathbf{v})$$

for all $\mathbf{u}, \mathbf{v}$ in the domain of T and for all scalars c, d.