Math 300

Section 1.5 Solution Sets of Linear Systems

A system of linear equations is homogeneous if its matrix equation is of the form $A\mathbf{x} = \mathbf{0}$ where A is an $m \times n$ matrix and $\mathbf{0}$ is the zero vector in $\mathbb{R}^m$.

Notes

1. The equation $A\mathbf{x} = \mathbf{0}$ always has the solution $\mathbf{x} = \mathbf{0}$. This solution is called the trivial solution. Thus a solution always exists for a homogeneous system.
2. The uniqueness question for a homogeneous system can be stated as follows: Does $A\mathbf{x} = \mathbf{0}$ have a non-trivial solution?

Theorem If the system of linear equations with matrix equation $A\mathbf{x} = \mathbf{b}$ is consistent and $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{b}$, then the solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is a solution of $A\mathbf{x} = \mathbf{0}$. Thus the solution set of $A\mathbf{x} = \mathbf{b}$ is a translation of the solution set of $A\mathbf{x} = \mathbf{0}$.