Math 300

Section 1.3 Vector Equations

A vector is a matrix with one column.

Algebraic Properties of $\mathbb{R}^n$

For all $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ in $\mathbb{R}^n$ and all scalars c and d :

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3. $\mathbf{u} + \mathbf{0} = 0 + \mathbf{u} = \mathbf{u}$
4. $\mathbf{u}$ + (-$\mathbf{u}$) = -$\mathbf{u}$ + $\mathbf{u}$ = $\mathbf{0}$ where -$\mathbf{u}$ denotes (-1)$\mathbf{u}$
5. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7. $c(d\mathbf{u}) = (cd)\mathbf{u}$
8. $1\mathbf{u} = \mathbf{u}$

If $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_p$ are vectors in $\mathbb{R}^n$ and $c_1, c_2, \cdots, c_p$ are scalars, then

$$\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_p\mathbf{v}_p$$

is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_p$ with weights $c_1, c_2, \cdots, c_p$.

If $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_p$ are vectors in $\mathbb{R}^n$, then the set of all possible linear combinations of $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_p$ is called the subset of $\mathbb{R}^n$ spanned by the set $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_p\}$ or the span of $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_p$. It is denoted Span$\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_p\}$.

The vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots x_n\mathbf{a}_n = \mathbf{b}$ has the same solution set as the system of linear equations with augmented matrix $[\mathbf{a}_1\ \mathbf{a}_2\ \cdots\ \mathbf{a}_n\ \mathbf{b}]$. The vector $\mathbf{b}$ is a linear combination of $\{\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_n\}$ (that is, $\mathbf{b}$ is in the Span$\{\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_n\}$) if and only if this system is consistent.

Three Ways to Ask the Same Question

1. Is $\mathbf{b}$ a linear combination of the columns of A?
2. Is $\mathbf{b}$ in Span$\{\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_n\}$?
3. Is the system with augmented matrix $[A\ \mathbf{b}]$ consistent?