Math 300

Section 1.1 Systems of Linear Equations

A linear equation in the variables $x_1, x_2, \cdots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where b and the coefficients $a_1, a_2, \cdots, a_n$ are real or complex numbers.

A system of linear equations is a collection of one or more linear equations involving the same variables.

A solution of the system is a list $(s_1, s_2, \cdots, s_n)$ of numbers that makes each equation a true statement when the values $s_1, s_2, \cdots, s_n$ are substituted for $x_1, x_2, \cdots, x_n$ respectively.

Two linear systems are called equivalent if they have the same solution set.

A system of linear equations is said to be consistent if it has either one solution or infinitely many solutions; a system is inconsistent if it has no solutions.

The essential information of a linear system can be recorded compactly in a rectangular array called a matrix. The matrix formed by the coefficients of a linear system is called the coefficient matrix and the matrix formed by adding the right hand side of a linear system an additional column to the coefficient matrix is called the augmented matrix.

The size of a matrix tells how many rows and columns it has. An $m \times n$ matrix has m rows and n columns.

Elementary Row Operations

(Replacement) Replace one row by the sum of itself and a multiple of another row.

(Interchange) Interchange two rows.

(Scaling) Multiply all entries in a row by a nonzero constant.

Two matrices are called row equivalent if there is a sequence of elementary row operations that transforms one matrix into the other.

Theorem If the augmented matrices of two linear systems are row equivalent, then the two systems have the same solution set.