Math 201 Calculus I

Course Activity: Antiderivatives and the First Fundamental Theorem of Calculus

Name: ___________________________________

Purpose: To begin investigating the relationship between a function and its antiderivatives – leading to the development of the First Fundamental Theorem of Calculus.

Procedure: Work on the following activity with 1-2 other students during class (but be sure to complete your own copy) and finish the exploration outside of class.

1. The amount (in *mg*) of a drug in the bloodstream at time t (in hours after the initial dosing) is given by the function, $Q(t)$. The rate of change of the amount, Q, as the drug leaves the bloodstream and passes into the urine is described by the differential equation:

$$\frac{dQ}{dt} = -42.88e^{-.536t}.$$

   a. Write a symbolic formula for the family of solution functions, $Q(t)$, that satisfy this differential equation.

   b. Assume a patient is initially given 100mg of this drug.

      i. Write a symbolic formula for the specific solution function, $Q(t)$, solving this **initial value problem** (i.e., the function that satisfies both the differential equation and the initial condition.)

      ii. For this patient, calculate how much of this drug remains in the patient's bloodstream after 4 hours.

      iii. For this patient, calculate the change in the amount of this drug in the patient's bloodstream between two hours and four hours after the initial dosing. Your answer should be negative. What is the meaning of the negative sign?

2. Imagine living in a large town where a number of people become initially infected with a flu (from a visitor to the town, perhaps). After this initial infection, the rate at which people are getting infected, $g(t)$, (measured in people per day) as a function of time t (measured in days since the initial infection) is given by:

$$g(t) = 40te^{-.05t}$$

   a. Suppose that $G_1(t)$ is an **antiderivative** of $g(t)$. (Notice that it is not obvious how to express $G_1(t)$ symbolically with a formula in terms of t. We will learn how to do that in a later module in this course and it is not needed for your work here.) Suppose also that $\boldsymbol{G_1(0) = 10}$.

      i. Describe what $G_1(t)$ represents in terms of this infection model (i.e., what is the input/output of G_1?). Include units in your answer.

      ii. Calculate an estimate for the value of $G_1(4)$ (by hand using your calculator).

      iii. Describe how you could improve on your estimate?

      iv. Write a mathematical expression giving the **exact** value of $G_1(4)$.

      v. What does $G_1(7) - G_1(4)$ represent?

      vi. Calculate an estimate for the value of $G_1(7) - G_1(4)$ (by hand using your calculator).

      vii. Describe how you could improve on your estimate?

viii. Write a mathematical expression giving the **exact** value of $G_1(7) - G_1(4)$.

b. Suppose that $G_2(t)$ is another **antiderivative** of $g(t)$. Suppose that $\boldsymbol{G_2(0) = 18}$.

i. Describe what $G_2(t)$ represents in terms of this infection model (i.e., what is the input/output of G_2?). Include units in your answer.

ii. Calculate an estimate for the value of $G_2(4)$ (by hand using your calculator).

iii. Describe how you could improve on your estimate?

iv. Write a mathematical expression giving the **exact** value of $G_2(4)$.

v. What does $G_2(7) - G_2(4)$ represent?

vi. Calculate an estimate for the value of $G_2(7) - G_2(4)$ (by hand using your calculator).

vii. Describe how you could improve on your estimate?

viii. Write a mathematical expression giving the **exact** value of $G_2(7) - G_2(4)$.

ix. Compare your answers in part (b) with their counterparts in part (a)? Explain the similarities and the differences you see.

3. Group Discussion: As a group, generalize the work that you did in problem #2 – which happened to be in terms of an infection model application where $g(t)$ represented the rate at which people in a town were getting infected by a flu.
   - ✓ Suppose that we have a general (continuous) function, $f(t)$.
   - ✓ Let $F(t)$ be an antiderivative of f. In other words, $F'(t) = f(t)$.

Conjecture the relationship that exists between the antiderivative, $F(t)$, and a definite integral of $f(t)$:

Explain your reasoning in forming this conjecture. (Your reasoning should mirror the thinking you employed in the special case of Problem #2.)

- ✓ The result you have uncovered gives us a very handy tool for easily computing the value of a definite integral without having to go through the process of obtaining better and better approximations (as is indicated by the definition of the definite integral).

- ✓ This result is called the (First) Fundamental Theorem of Calculus: If a function $f(t)$ is continuous on the interval $[a, b]$ and $F'(t) = f(t)$, then:

Methods Practice: Complete the following question individually.

1. **Analytical/Symbolic**: Evaluate the following definite integrals using the Fundamental Theorem of Calculus:

   a. $\int_1^4 (x^2 + 4x + 3)dx =$

   b. $\int_2^5 \frac{1}{t}\, dt =$

   c. $\int_0^2 e^{3x} dx =$

   d. $\int_{-2}^3 \frac{2}{r^3}\, dr =$

   (Are the conditions of the FTC really satisfied in number 4?)

2. **Narrative**: Answer the following narrative questions using the Fundamental Theorem of Calculus:

   a. The rate at which the world's oil is being consumed (in billions of barrels per year) is given by the function, $r(t) = 32e^{0.05t}$, where t is measured in years and $t = 0$ is the start of 1990. Find the total quantity of oil used between the start of 1990 and the start of 1995.

   b. Suppose on a recent car trip your velocity (in miles per hour) was given by the function, $v(t) = -15\,t^4 + 96\,t^3 - 219.6\,t^2 + 211.2\,t$, where t is measured in hours. Your trip lasted from $t = 0$ to $t = 3.2$ hours.

      a. Find the total distance travelled by you on this trip.

      b. Find your average velocity on this trip.