Math 201 Calculus I

Module 1: Average versus Instantaneous Speed

*Name*: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*Purpose*: To explore the relationship between average and instantaneous speed, and then to begin to generalize these ideas to different contexts.

*Procedure*: Work on the following activity together as a class, then complete outside of class.

**Speed Cameras:** Many locations around the world are now using **traffic enforcement cameras** – which include cameras and vehicle-monitoring devices to detect and identify vehicles disobeying speed limits. **Speed Cameras** are used to identify vehicles traveling over the legal speed limit. Cameras are installed on bridges above traffic lanes, for example, in two different locations along a roadway. Each camera automatically captures an image of the vehicle and a computer identifies and recognizes the license plate of the passing vehicle. The pair of images taken of a single vehicle are matched by the license plate and speed is calculated.

The picture below is a sample pair of such images. The two cameras that took these images are located at a distance of 0.2 miles apart on this particular roadway. The times printed in the upper left hand corner of each image are difficult to read. They are: (1) 12:03:52 and (2) 12:04:01.

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Questions:

1. How fast is the car pictured above traveling? Explain your reasoning.
2. When is the car traveling at that speed?
3. Is it fair to give the driver of this car a traffic ticket for speeding if the speed you calculated exceeds the speed limit? Why or why not?
4. The speed calculation you made above was for **average speed** (over the time interval from 12:03:52 to 12:04:01). Exactly how fast is the car traveling *at the instant*, 12:03:52? Is it possible to directly measure **instantaneous speed** or do you need additional information? – if so, what additional information would help?

Consider a similar situation where cameras are located at 10 locations along a roadway and the following information about a passing car is captured:

|  |  |
| --- | --- |
| *Time (mins past start camera)* | *Distance (miles from start camera)* |
| 0.5 | 0.06 |
| 1.0 | 0.1 |
| 1.5 | 0.16 |
| 2.0 | 0.28 |
| 2.5 | 0.47 |
| 3.0 | 0.79 |
| 3.5 | 1.33 |
| 4.0 | 2.23 |
| 4.5 | 3.74 |
| 5.0 | 6.27 |

We would now like to answer the question: How fast is this car traveling at the exact instant 2.7 minutes past the start camera? Provide a strategy for obtaining your most accurate estimate to answer the above question. Carry out your strategy – working with a partner. Explain all of your work. Provide a graphical representation of this estimate.

Find a function that models the distance data provided above. Using this model, calculate another estimate to answer the question, “How fast is this car traveling at the exact instant 2.7 minutes past the start camera?”. Provide a graphical representation of this estimate.

Discussion:

* The quantity you estimated above is called **instantaneous speed**. How would you find a *better estimate* of this car’s instantaneous speed at exactly 2.7 minutes past the start camera?
* The process of obtaining better and better estimates for instantaneous speed implies that instantaneous speed can be described as a ***limiting value***. Write down a definition for **instantaneous speed** (involving a limit statement):

Glucose in Bloodstream: A physician decides to give a patient an infusion of glucose at a rate of $10$ grams per hour. The body of the patient simultaneously converts the glucose and removes it from the bloodstream at a rate proportional to the amount present in the bloodstream – specifically at a rate of 3 grams per hour per gram of glucose present.

Let $G=G(t)$ represent the amount of glucose present in the bloodstream at time $t$ hours after infusion begins. This function can be described by the following symbolic formula: $G\left(t\right)=\frac{10}{3}-0.63e^{-3t}$.

* What is $G(0.1)$? What is the meaning of $G(0.1)$ in the context of this application?
* What is $G(0.5)$? What is the meaning of $G(0.5)$ in the context of this application?
* What is the rate of change of the amount of glucose present in the bloodstream from $t=0.1 hours$ to $t=0.5 hours$ (include units)? (Because this rate of change is *between two points*, we will call it the **average rate of change** of $G(t)$ from $t=0.1$ to $t=0.5$.)
* *Estimate* how fast the amount of glucose in the bloodstream is changing at the instant, $t=0.5$ hours after infusion begins (include units). Keep working on your estimate until you are accurate to within two decimal places. (Because this rate of change is at an instant, we will call it the **instantaneous rate of change** of $G(t)$ at $t=0.5$.)
* Estimate the instantaneous rates of change of $G\left(t\right)$ at two other instants:

	+ At $t=0.8$ hours:
	+ At $t=1.0$ hours:

Discussion: For a general function, $f(x)$, and constants $x\_{0}, x\_{1}, and x\_{2}$:

* The **Average Rate of Change** of $f(x)$ from $x=x\_{1}$ to $x=x\_{2}$ is defined by:
* The **Instantaneous Rate of Change** of $f(x)$ at $x=x\_{0}$ is defined by: