Write-Up 1.5 Pg 100 Ex 1.5

(47) Show that the equation $x^3+x^2-2x=1$ has at least one solution in the interval $[-1,1]$

First let's subtract a 1 from both sides

$$\underbrace{x^3+x^2-2x-1}=0$$

Let this polynomial define the function $f(x)$ such that $f(x)=x^3+x^2-2x-1$

On a graph the solutions of $f(x)$ are where the function croses the x axis or where $f(x)=0$

The IVT tells us that a graph of a polynomial above the x axis for one value and below the x axis for another value of x must cross the x axis somewhere between the two different values of x.

We want to show that $f(x)$ has at least one solution in the interval $[-1,1]$

Let's consider the two extremes in the interval $[-1, 1]$

$f(x) = x^3 + x^2 - 2x - 1$

$f(-1) = -1 + 1 + 2 - 1 = 1$

$f(-1) > 0$ and is therefore above the x axis

$f(1) = 1 + 1 - 2 - 1 = -1$

$f(1) < 0$ and is therefore below the x axis

If $f(x)$ is continuous and has a point above the x axis and a point below the x axis, it must cross the x axis. The value of x when $f(x)$ crosses the x axes is a solution.

$\therefore$ the equation $x^3 + x^2 - 2x = 1$ has a solution.

Fantastic! I really like that you explained what the IVT does in your own words. I think that anyone who reads through this explanation will have a better understanding of how to solve problems like these and the IVT in general. Great job!