The Joy of Pi

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3.14

Obsession with Pi

People with time on their hands

Pi in Song

Search Pi

It All Began...

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However, the first trace of a calculation of π can be found in the Egyptian Rhind Papyrus, which is dated about 1650 BC and gives $\pi = 4\left(\frac{8}{9}\right)^2 = 3.16$.

A Little History

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Euler adopted the symbol π in 1737 (after it was introduced by William Jones in 1706) and it quickly became standard notation.

Buffon's Needle Experiment

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Various people have tried to calculate π by throwing needles. The most remarkable result was that of Lazzerini (1901), who made 34080 tosses and got

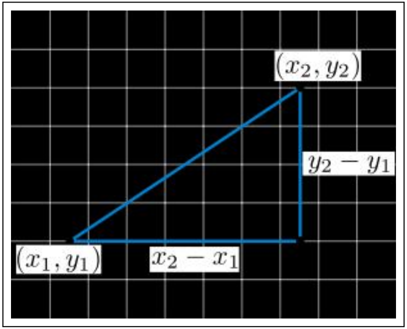
$$\pi = \frac{335}{113} = 3.1415929.$$

Some open questions about the number π

Does each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 each occur infinitely often in π?

Brouwer's question: In the decimal expansion of π, is there a place where a thousand consecutive digits are all zero?

Where does our distance formula for $\mathbb{R}^2$ come from?



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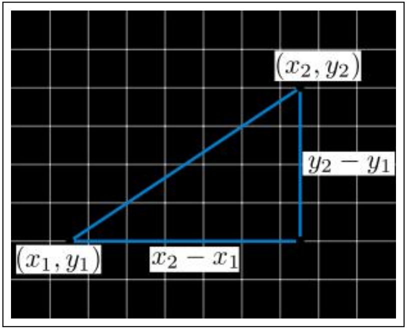
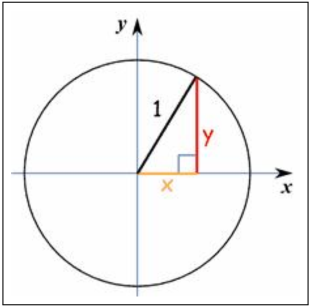


By Pythagoras' Theorem,

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

What does this have to do with the unit circle?



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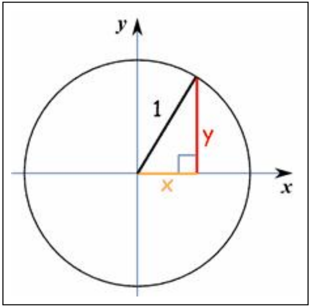

The unit circle is all points (x, y) that are one-unit distance from the origin; ie,

$$1 = \sqrt{(x-0)^2 + (y-0)^2}$$
$$1 = x^2 + y^2$$

What's in a norm?

Given a vector space V over a subfield F of the complex numbers, a **norm** on V is a function $\| \cdot \| : V \to F$ with the following properties:
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$\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$ (Triangle Inequality).

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The Taxicab norm (or Manhattan norm) for $\mathbb{R}^n$ is

$$\| < x_1, x_2, \cdots, x_n > \|_1 = |x_1| + |x_2| + \cdots |x_n|.$$

Back to our idea of distance in $\mathbb{R}^2$

What if we used the Taxicab norm (instead of the Euclidean norm) to compute the distance between two points?

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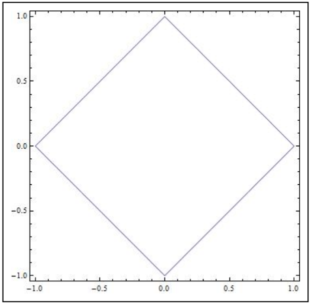
"Unit circle"



To infinity and beyond!

Let's define "*p*-distance" as

$$d_p((x_1, y_1), (x_2, y_2)) = (|x_2 - x_1|^p + |y_2 - y_1|^p)^{1/p} = \|x_2 - x_1\|_p$$

where $x_1 = (x_1, y_1)$ and $x_2 = (x_2, y_2)$. We can now visualize what the unit ball (ie, the set of all points of distance 1 from the origin) looks like under these different norms in $\mathbb{R}^2$.

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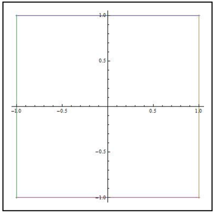

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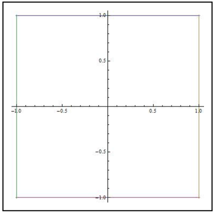

We define

$$d_\infty((x_1, y_1), (x_2, y_2)) = \max\{|x_2 - x_1|, |y_2 - y_1|\}.$$

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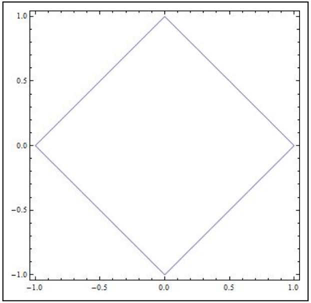

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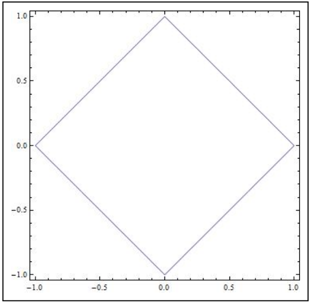

$d((1,0),(0,1))_1 = |0-1| + |1-0| = 2$ so $C = 8$ and $r = 1$ which gives $\pi = 4!$

What?!

From Calculus, we can compute the length of the curve $y = f(t)$ from $t = a$ to $t = b$ using a formula for arc length:

$$L = \int_a^b \sqrt{1 + [f'(t)]^2} dt.$$

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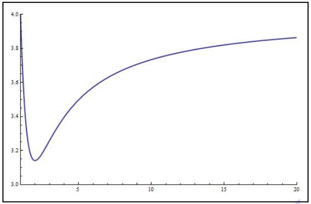

Thank you for having me!!