**Section 3.5** Linearization of Nonlinear Systems

**Theorem** Let $f(x) = A(x - x_0) + g(x)$ where $\lim_{x \to x_0} \frac{\|g(x)\|}{\|x - x_0\|} = 0$.

(i) If all eigenvalues of $A$ have negative real parts, then the equilibrium $x_0$ is asymptotically stable.

(ii) If some eigenvalue of $A$ has positive real part, then $x_0$ is unstable.

**Theorem** In the case of two equations in two unknowns, assume $A$ has eigenvalues $\lambda_1, \lambda_2$ with $\lambda_1 < 0 < \lambda_2$. Then there are two orbits of $x' = f(x)$ that go to $x_0$ as $t \to \infty$ along a smooth curve tangent at $x_0$ to the eigenvectors for $\lambda_1$ and two orbits that go to $x_0$ as $t \to -\infty$ along a smooth curve tangent at $x_0$ to the eigenvectors of $\lambda_2$.