Section 1.2 First-Order Linear Equations

Variation of Constants Formula If $p : (a, b) \to \mathbb{R}$ and $q : (a, b) \to \mathbb{R}$ are continuous functions, where $-\infty \le a < b \le \infty$, then the unique solution x of the IVP

$$x' = p(t)x + q(t), \quad x(t_0) = x_0,$$

where $t_0 \in (a, b), x_0 \in \mathbb{R}$, is given by

$$x(t) = e^{\int_{t_0}^{t} p(\tau)d\tau} x_0 + e^{\int_{t_0}^{t} p(\tau)d\tau} \int_{t_0}^{t} e^{-\int_{t_0}^{s} p(\tau)d\tau} q(s)ds,$$

$t \in (a, b)$.

Newton's Law of Cooling Newton's law of cooling states that the rate of change of the temperature of an object is proportional to the difference between its temperature and the temperature of the surrounding medium. That is,

$$x' = k(x - T)$$

where $x(t)$ is the temperature of the object at time t, $T(t)$ is the temperature of the surrounding medium at time t, and k is the constant of proportionality.