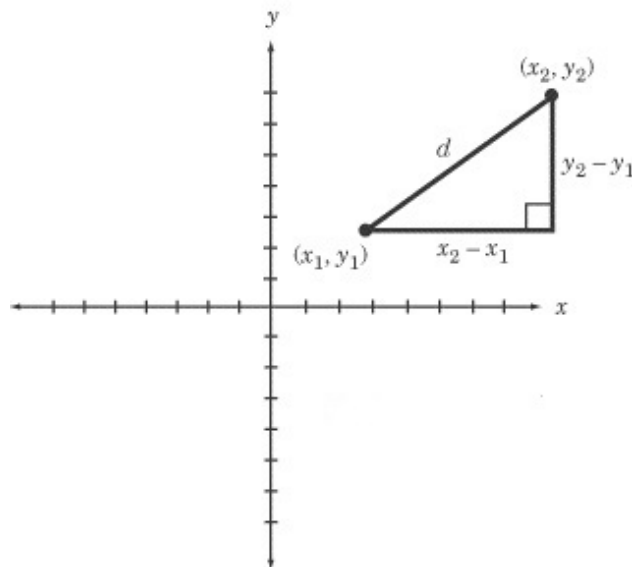


## Math 509 Project 1: $\pi = \text{What?!}$

The purpose of this project is to explore various ways to measure distance in the  $xy$ -plane. We've seen the importance of distance in class, namely using absolute value to measure the closeness between sequences and their limits. On the real number line, we use the absolute value function to measure distance, thinking of  $|x|$  as the distance from  $x$  to 0. On  $\mathbb{R} \times \mathbb{R}$ , or the  $xy$ -plane, the common way we measure distance is called Euclidean distance (after the famous geometer Euclid):

$$\text{distance between } (x_1, y_1) \text{ and } (x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

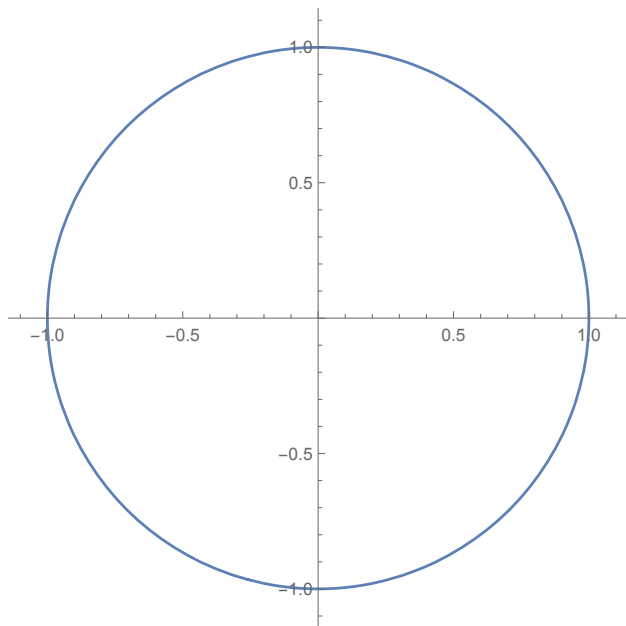
This distance formula is derived from Pythagorean theorem, thinking of the distance between two points as a straight line:



The distance formula plays an important role in the calculation of the unit circle. By definition, the unit circle is the set of all points that are distance one from the origin. If we write this mathematically, we get:

$$\begin{aligned} 1 &= \sqrt{(x - 0)^2 + (y - 0)^2} \implies \\ 1 &= \sqrt{x^2 + y^2} \implies \\ 1 &= x^2 + y^2. \end{aligned}$$

Graphically, this is depicted by:



There are other common methods for calculating distance in the  $xy$ -plane. Two important methods are the taxicab distance:

$$\text{taxicab distance between } (x_1, y_1) \text{ and } (x_2, y_2) = |x_2 - x_1| + |y_2 - y_1|$$

and the max-distance:

$$\text{max-distance between } (x_1, y_1) \text{ and } (x_2, y_2) = \max\{|x_2 - x_1|, |y_2 - y_1|\}.$$

In general, we define  $p$ -distance as:

$$p\text{-distance between } (x_1, y_1) \text{ and } (x_2, y_2) = \sqrt[p]{(x_2 - x_1)^p + (y_2 - y_1)^p}.$$

It might be helpful to note that when  $p = 1$  we get the taxicab distance and when  $p \rightarrow \infty$  we approach the max-distance.

**Task 1** Draw the unit circle in the taxicab distance and the max distance. What common shapes are these?

**Task 2** Choose any real number  $p$ ,  $1 < p < \infty$  and draw the unit circle in this  $p$ -distance. (You'll find it helpful to use *Mathematica's* ContourPlot command!)

It turns out that the value of  $\pi$  is dependent on our choice of formula for distance.  $\pi$  is defined as the constant ratio of circumference and diameter for any circle:

$$\pi = \frac{C}{d}.$$

**Task 3** Calculate the value of  $\pi$  in the taxicab and max distance. Remember to use the correct calculation of distance when calculating the circumference of your unit circles!

**Task 4** Modify the arc length formula from Calculus 2 to calculate the value of  $\pi$  for your choice of  $p$  in Task 2. You'll want to use *Mathematica's* NIntegrate command to do this!

If you plot  $p$  versus the value of  $\pi$  in  $p$ -distance, you'll see that the Euclidean value of  $\pi$  (3.14159265358979323...) is the minimum value of  $\pi$ . This gives some context to the saying "The shortest distance between two points is a straight line."