Name:__________________________________

Math 311 Fall 2017

Project 2 - Due 12/4

Introduction

In this project, we view a sequence $\{a_n\}$ as a function $a : \mathbb{Z} \to \mathbb{R}$ which associates to every integer $n \in \mathbb{Z}$ the real number $a_n \in \mathbb{R}$. Thus, for example, if $a_n = n^3$ then some of the values of a are:

n:	$\cdots$	-3	-2	-1	0	1	2	3	$\cdots$
$a_n = n^3$:	$\cdots$	-27	-8	-1	0	1	8	27	$\cdots$

We define the *forward difference operator* Δ_+ as the function which associates to each sequence a the sequence $\Delta_+ a$ whose nth term is

$$(\Delta_+ a)_n = a_{n+1} - a_n,$$

the *backward difference operator* Δ_- as the function which associates to each sequence a the sequence $\Delta_- a$ whose nth term is

$$(\Delta_- a)_n = a_n - a_{n-1},$$

and the *sum operator* Σ as the function which associates to each sequence a the sequence Σa whose nth term is

$$(\Sigma a)_n = \sum_{i=1}^{n} a_i.$$

Questions

1. If

$$a_n = \begin{cases} 0 & \text{if } n < 0 \\ \frac{1}{2^n} & \text{if } n \geq 0, \end{cases}$$

find, in simplest form: a) $(\Delta_+ a)_n$, b) $(\Delta_- a)_n$, and c) $(\Sigma a)_n$.

2. Let a and b be sequences and c a constant. Prove:

a) $\Delta_+(a + b) = \Delta_+(a) + \Delta_+(b)$

b) $\Delta_+(ca) = c\Delta_+(a)$

c) $\Delta_-(a + b) = \Delta_-(a) + \Delta_-(b)$

d) $\Delta_-(ca) = c\Delta_-(a)$

Based on these assertions, what type of operator is Δ_+ and Δ_-?

3. Derive and prove formulas for the following:

a) $\Sigma 1$

b) Σn

c) Σn^2

4. What:

a) is the composition $\Sigma \circ \Delta_+$? That is, what is the value of

$$((\Sigma \circ \Delta_+)a)_n$$

for any sequence a, in simplest form? How about $\Sigma \circ \Delta_-$?

b) is the composition $\Delta_+ \circ \Sigma$? How about $\Delta_- \circ \Sigma$?

c) result from Calculus I do parts (a) and (b) remind us of?

5. Use the results from #4 to establish solutions to the following difference equations:

a) $a_n = a_{n-1}$ with initial condition $a_0 = a(0)$.

b) $a_n = a_{n-1} + c$ with initial condition $a_0 = a(0)$, where c is a constant.

c) $a_n = a_{n-1} + n$ with initial condition $a_0 = a(0)$.