Section 15.1 Vector Fields

Let D be a set in $\mathbb{R}^2$ (a plane region). A vector field on $\mathbb{R}^2$ is a function $\mathbf{F}$ that assigns to each point (x, y) in D a two-dimensional vector $\mathbf{F}(x, y)$.

If f is a scalar function of two variables, recall that its gradient ∇f is defined by

$$\nabla f(x, y) = f_x(x, y)\mathbf{i} + f_y(x, y)\mathbf{j}.$$

Therefore, ∇f is really a vector field on $\mathbb{R}^2$ and is called a gradient vector field.

A vector field $\mathbf{F}$ is called a conservative vector field if it is the gradient of some scalar function, that is, if there exists a function f such that $\mathbf{F} = \nabla f$. In this situation, f is called a potential function for $\mathbf{F}$.