Section 14.3 Double Integrals in Polar Coordinates

To change from rectangular coordinates to polar coordinates, use the equations:

$$x = r\cos(\theta) y = r\sin(\theta)$$

Change to Polar Coordinates in a Double Integral If f is continuous on a polar rectangle R given by $0 \leq a \leq r \leq b$, $\alpha \leq \theta \leq \beta$, where $0 \leq \beta - \alpha \leq 2\pi$, then

$$\iint\limits_R f(x,y)dA = \int_\alpha^\beta \int_a^b f(r\cos(\theta), r\sin(\theta))rdrd\theta.$$

If f is continuous on a polar region of the form

$$D = \{(r,\theta)|\alpha \leq \theta \leq \beta, h_1(\theta) \leq r \leq h_2(\theta)\}$$

then

$$\iint\limits_D f(x,y)dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos(\theta), r\sin(\theta))rdrd\theta.$$