Section 14.1 Double Integrals

The double integral of f over the rectangle R is

$$\iint_R f(x,y)dA = \lim_{m,n\to\infty} \sum_{i=1}^{n}\sum_{j=1}^{m} f(x_{ij}*, y_{ij}*)\Delta A$$

Theorem If $f(x,y) \geq 0$, then the volume V of the solid that lies above the rectangle R and below the surface $z = f(x,y)$ is

$$V = \iint_R f(x,y)dA.$$

Properties of Double Integrals

1. $\iint_R [f(x,y) + g(x,y)]dA = \iint_R f(x,y)dA + \iint_R g(x,y)dA.$
2. $\iint_R cf(x,y)dA = c\iint_R f(x,y)dA$ where c is a constant.
3. If $f(x,y) \geq g(x,y)$ for all $(x,y) \in R$, then $\iint_R f(x,y)dA \geq \iint_R g(x,y)dA.$

Iterated Integrals

$$\int_a^b \int_c^d f(x,y)dydx = \int_a^b \left[\int_c^d f(x,y)dy\right] dx$$

means to first integrate with respect to y (ie, hold x fixed); then plug in constants c and d for y. Once you've done this, you'll be left with a function you can integrate with respect to x. Work inside-out.

$$\int_c^d \int_a^b f(x,y)dxdy = \int_c^d \left[\int_a^b f(x,y)dx\right] dy.$$

Fubini's Theorem If f is continuous on the rectangle $R = \{(x,y)|a \leq x \leq b, c \leq y \leq d\}$, then

$$\iint_R f(x,y)dA = \int_a^b \int_c^d f(x,y)dydx = \int_c^d \int_a^b f(x,y)dxdy.$$