Section 13.3 Partial Derivatives

If f is a function of two variables, its partial derivatives are the functions f_x and f_y defined by

$$f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$$

$$f_y(x,y) = \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h}$$

Theorem $f_x(a,b) = g'(a)$ where $g(x) = f(x,b)$.

Notations for Partial Derivatives: If $z = f(x,y)$, we write

$$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}[f(x,y)] = \frac{\partial z}{\partial x} = D_x f$$

$$f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}[f(x,y)] = \frac{\partial z}{\partial y} = D_y f$$

Rule for Finding Partial Derivatives of $z = f(x,y)$

1. To find f_x, regard y as a constant and differentiate $f(x,y)$ with respect to x.
2. To find f_y, regard x as a constant and differentiate $f(x,y)$ with respect to y.

Note: Partial derivatives can also be interpreted as rates of change. If $z = f(x,y)$, then $\dfrac{\partial z}{\partial x}$ represents the rate of change of z with respect to x when y is fixed. Similarly, $\dfrac{\partial z}{\partial y}$ represents the rate of change of z with respect to y when x is fixed.

Higher Derivatives

If f is a function of two variables, then its partial derivatives f_x and f_y are also functions of two variables, so we can consider the second partial derivatives of f : $f_{xx}, f_{xy}, f_{yx}, f_{yy}$.

Clairaut's Theorem Suppose f is defined on a disk D that contains the point (a,b). If the functions f_{xy} and f_{yx} are both continuous on D, then $f_{xy}(a,b) = f_{yx}(a,b)$.