Section 13.2 Limits and Continuity

We write

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

and we say that the limit of $f(x, y)$ as (x, y) approaches (a, b) is L if we can make the values of $f(x, y)$ as close to L as we like by taking the point (x, y) sufficiently close to the point (a, b) but not equal to (a, b).

Theorem If $f(x, y) \to L_1$ as $(x, y) \to (a, b)$ along a path C, and $f(x, y) \to L_2$ as $(x, y) \to (a, b)$ along a path C_2, where $L_1 \neq L_2$, then $\lim\limits_{(x,y)\to(a,b)} f(x, y)$ does not exist.

Note: The Limit Laws from Calculus I hold (ie, the limit of the sum is the sum of the limits, the limit of a constant is that constant, the Squeeze Theorem).

Continuity A function f of two variables is called continuous at (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b).$$

We say f is continuous on D if f is continuous at every point (a, b) in D.