

## Section 12.5 Curvature

The curvature of a curve is

$$\kappa = \left| \frac{d\mathbf{T}}{ds} \right|$$

where  $\mathbf{T}$  is the unit tangent vector.

By Chain Rule,  $\frac{d\mathbf{T}}{dt} = \frac{d\mathbf{T}}{ds} \frac{ds}{dt}$  so

$$\kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \left| \frac{d\mathbf{T}/dt}{ds/dt} \right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}.$$

**Theorem**  $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ .

**Theorem** For the special case of a plane curve with equation  $y = f(x)$ , we have

$$\kappa(x) = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}}$$