Section 12.2 Calculus of Vector-Valued Functions

Derivatives If $\mathbf{r}(t) =< f(t), g(t), h(t) >$, where f, g and h are differentiable functions, then

$$\mathbf{r'}(t) =< f'(t), g'(t), h'(t) > .$$

The unit tangent vector $\mathbf{T}(t)$ is given by

$$\mathbf{T}(t) = \frac{\mathbf{r'}(t)}{|\mathbf{r'}(t)|}$$

Differentiation Rules Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable vector functions, c is a scalar, and f is a real-valued function. Then

1. $\frac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \mathbf{u'}(t) + \mathbf{v'}(t)$
2. $\frac{d}{dt}[c\mathbf{u}(t)] = c\mathbf{u'}(t)$
3. $\frac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u'}(t)$
4. $\frac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u'}(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v'}(t)$
5. $\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u'}(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v'}(t)$
6. $\frac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\mathbf{u'}(f(t))$

Integrals If $\mathbf{r}(t) =< f(t), g(t), h(t) >$, then

$$\int_a^b \mathbf{r}(t)dt = \left(\int_a^b f(t)dt\right)\mathbf{i} + \left(\int_a^b g(t)dt\right)\mathbf{j} + \left(\int_a^b h(t)dt\right)\mathbf{k}.$$