

Section 12.1 Introduction to Vector-Valued Functions

A vector-valued function, or vector function, is a function whose domain is a set of real numbers and whose range is a set of vectors:

$$\begin{aligned}\mathbf{r}(t) &= \langle f(t), g(t), h(t) \rangle \\ &= f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}\end{aligned}$$

Theorem If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, then $\lim_{t \rightarrow a} \mathbf{r}(t) = \left\langle \lim_{t \rightarrow a} f(t), \lim_{t \rightarrow a} g(t), \lim_{t \rightarrow a} h(t) \right\rangle$ provided the limits of the component functions exist.

Corollary A vector function \mathbf{r} is continuous at a if $\lim_{t \rightarrow a} \mathbf{r}(t) = \mathbf{r}(a)$.